Exercise 21:

Given is the following difference equation

$$g(k) = 0.1 \cdot s(k) - 0.9 \cdot g(k-1) \qquad \text{with} \qquad s(k) = 0 \;\; \forall \;\; k < 0$$

of a digital filter. Before excitation the system was in zero state.

1. Draw the structure of the filter.

2. Is it a recursive or non-recursive system. Justify your answer.

3. Determine the system function $H_{\mathrm{z}}(z)$ of the filter.

4. Determine the poles and zeros of $H_{\mathrm{z}}(z)$.

5. At the input of the digital filter the sequences $s(k) = \gamma_0(k)$ (Case 1), and $s(k) = \gamma_{-1}(k)$ (Case 2) are applied. Enlist for each case in the range of $k = 0(1)5$ the values of $0.1 \cdot s(k)$, $-0.9 \cdot g(k-1)$ and $g(k)$.

6. Determine the impulse response $h(k)$ from the system function $H_{\mathrm{z}}(z)$. In the range $k = 0(1)5$ calculate the values of $h(k)$ and compare them with the results of 21.5.

7. Determine the back-transformed $h_{\mathrm{a}}(t)$ of $H_{\mathrm{aL}}(p)$ and determine $H_{\mathrm{aL}}(p) = H_{\mathrm{z}}(z = \mathrm{e}^{pT_{\mathrm{a}}})$, with T_{a} the sampling period.