NT/UNI Duisburg	**Auxiliary leaflet** Lecture and exercises "Communication Engineering"	**11** 12/98

Laplace transform: Theorems and properties

1	Linearity	$s_1(t) \circ\!\!-\!\!\bullet F_1(p)$, $s_2(t) \circ\!\!-\!\!\bullet F_2(p)$; $[a_1 \cdot s_1(t) + a_2 \cdot s_2(t)] \circ\!\!-\!\!\bullet a_1 \cdot F_1(p) + a_2 \cdot F_2(p)$
2	Similarity, scaling-theorem	$s(t) \circ\!\!-\!\!\bullet F(p)$ $s(b \cdot t) \circ\!\!-\!\!\bullet \frac{1}{b} \cdot F\left(\frac{p}{b}\right)$ b, c -real and positive $F(c \cdot p) \bullet\!\!-\!\!\circ \frac{1}{c} \cdot s\left(\frac{t}{c}\right)$
3	Translation rules (Attenuation)	$s(t) \circ\!\!-\!\!\bullet F(p)$ $\underline{\alpha > 0:}$ $s(t-\alpha) \circ\!\!-\!\!\bullet F(p) \cdot e^{-\alpha p}$ $s(t+\alpha) \circ\!\!-\!\!\bullet [F(p) - \int_o^\alpha s(t) \cdot e^{-pt} \cdot dt] \cdot e^{\alpha p}$ p_0 arbitrary $\rightarrow$ $e^{-p_0 t} \cdot s(t) \circ\!\!-\!\!\bullet F(p+p_0)$
4	Differentiation in the time domain	$s^{(n)}(t) \circ\!\!-\!\!\bullet p^n \cdot F(p) - s(+0) \cdot p^{n-1} - s'(+0) \cdot p^{n-2} - \ldots - s^{n-1}(+0)$
5	Differentiation in the Laplace domain	for all p , $F^{(n)}(p) \bullet\!\!-\!\!\circ (-1)^n \cdot t^n \cdot s(t)$ with $F(p)$ describing a regular function
6	Integration in the time domain	$\int_0^t s(\tau) \cdot d\tau \circ\!\!-\!\!\bullet \frac{1}{p} \cdot F(p)$
7	Integration in the Laplace domain	$\int_p^\infty F(q) \cdot dq \bullet\!\!-\!\!\circ \frac{1}{t} \cdot s(t)$
8	Convolution, multiplication	$s_1(t) \circ\!\!-\!\!\bullet F_1(p)$, $s_2(t) \circ\!\!-\!\!\bullet F_2(p)$ $F_1(p) \cdot F_2(p) \bullet\!\!-\!\!\circ \int_0^t s_1(\tau) \cdot s_2(t-\tau) \cdot d\tau = s_1(t) * s_2(t)$
9	Periodicity	Impulse sequence: $s(t) = s(t - n \cdot T)$; $s(t) = 0$ für $t < 0$, $n = 0, 1, \ldots, \infty$ $s(t) \circ\!\!-\!\!\bullet \frac{1}{1-e^{-pT}} \cdot \int_0^T s(t) \cdot e^{-pT} \cdot dt$
10	Limit theorems	$\lim_{p\to\infty} p \cdot F(p) = \lim_{t\to 0} s(t) = s(0)$; $s(0)$ must exist $\lim_{p\to 0} p \cdot F(p) = \lim_{t\to\infty} s(t) = s(\infty)$; $s(\infty)$ must exist