NT/UNI Duisburg	**Auxiliary leaflet** Lecture and exercises “Communication Engineering”	**9a** 12/98

Fourier transform: Theorems and properties

$$S(\omega) = \int_{-\infty}^{\infty} s(t) \cdot e^{-j\omega t} \cdot dt \qquad s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) \cdot e^{j\omega t} \cdot d\omega$$

$s_1(t) = Re\{s(t)\}$ $\quad s_2(t) = Im\{s(t)\}$ $\quad R(\omega) = Re\{S(\omega)\}$ $\quad X(\omega) = Im\{S(\omega)\}$

$s(t) = s_g(t) + s_u(t)$ with $s_g(t) = s_g(-t)$ and $s_u(t) = -s_u(-t)$

a, a_1, a_2 arbitrary constants

b, c, ω_0, t_0 real constants

1	$s(t) = s_1(t) + js_2(t)$	$R(\omega) = \int_{-\infty}^{\infty} [s_1(t) \cdot \cos(\omega \cdot t) + s_2(t) \cdot \sin(\omega \cdot t)] \cdot dt$ $X(\omega) = -\int_{-\infty}^{\infty} [s_1(t) \cdot \sin(\omega \cdot t) - s_2(t) \cdot \cos(\omega \cdot t)] \cdot dt$
2	$s(t)$ real $\Rightarrow s_2(t) \equiv 0$	$R(\omega) = R(-\omega)$ even function $X(\omega) = -X(-\omega)$ odd function $S^*(\omega) = S(-\omega) \quad ; \quad \lvert S(\omega)\rvert = \lvert S(-\omega)\rvert$
3	$s(t)$ imaginary $\Rightarrow s_1(t) \equiv 0$	$R(\omega) = -R(-\omega)$ odd function $X(\omega) = X(-\omega)$ even function $S(-\omega) = -S^*(\omega) \quad ; \quad \lvert S(\omega)\rvert = \lvert S(-\omega)\rvert$
4	$s(t)$ real and even $\rightarrow s_1(t) = s_1(-t); s_2(t) \equiv 0$	$R(\omega) = 2\int_{0}^{\infty} s_1(t) \cdot \cos(\omega \cdot t) \cdot dt$ $X(\omega) \equiv 0$
5	$s(t)$ real and odd $\rightarrow s_1(t) = -s_1(-t); s_2(t) \equiv 0$	$R(\omega) \equiv 0$ $X(\omega) = -2\int_{0}^{\infty} s_1(t) \cdot \sin(\omega \cdot t) \cdot dt$
6	$s(t)$ real $s_g(t) = \frac{1}{2} \cdot [s(t) + s(-t)]$ $s_u(t) = \frac{1}{2} \cdot [s(t) - s(-t)]$	$R(\omega) = 2\int_{0}^{\infty} s_g(t) \cdot \cos(\omega \cdot t) \cdot dt$ $X(\omega) = -2\int_{0}^{\infty} s_u(t) \cdot \sin(\omega \cdot t) \cdot dt$ $s_g(t) = \frac{1}{\pi}\int_{0}^{\infty} R(\omega) \cdot \cos(\omega \cdot t) \cdot d\omega$ $s_u(t) = -\frac{1}{\pi}\int_{0}^{\infty} X(\omega) \cdot \sin(\omega \cdot t) \cdot d\omega$
7	Linearity	$s_1(t) \circ\!\!-\!\!\bullet S_1(\omega)$ $s_2(t) \circ\!\!-\!\!\bullet S_2(\omega)$ $a_1 \cdot s_1(t) + a_2 \cdot s_2(t) \circ\!\!-\!\!\bullet a_1 \cdot S_1(\omega) + a_2 \cdot S_2(\omega)$