Fourier transform: Theorems and properties

8	Similarity, scaling theorem	$s(t) \circ\!\!-\!\!\bullet\ S(\omega)$	$s(b \cdot t) \circ\!\!-\!\!\bullet\ \frac{1}{\lvert b\rvert} \cdot S\left(\frac{\omega}{b}\right)$ $S(c \cdot \omega) \bullet\!\!-\!\!\circ\ \frac{1}{\lvert c\rvert} \cdot s\left(\frac{t}{c}\right)$
9	Shifting	$s(t-t_0)$ $S(\omega-\omega_0)$	$s(t-t_0) \circ\!\!-\!\!\bullet\ S(\omega) \cdot e^{-j\omega t_0}$ $S(\omega-\omega_0) \bullet\!\!-\!\!\circ\ s(t) \cdot e^{+j\omega_0 t}$
10	Differentiation in the time domain	$s(t)$ differentiable n times and $S(\omega)$ exists and $\lim\limits_{t\to\pm\infty} s^{(\nu)}(t) = 0$ for $\nu = 0, 1, \ldots, (n-1)$	$\frac{d^n s(t)}{dt^n} \circ\!\!-\!\!\bullet\ (j\omega)^n \cdot S(\omega)$
11	Differentiation in the frequency domain	$S^{(n)}(\omega)$ exists	$\frac{d^n S(\omega)}{d\omega^n} \bullet\!\!-\!\!\circ\ (-jt)^n \cdot s(t)$
12	Integration in the time domain	If $g(t) = \int\limits_{-\infty}^{t} s(\tau) \cdot d\tau$ absolutely integrable	$\int\limits_{-\infty}^{t} s(\tau) \cdot d\tau \circ\!\!-\!\!\bullet\ \frac{1}{j\omega} \cdot S(\omega)$ for $S(0) = 0$ $\int\limits_{-\infty}^{t} s(\tau) \cdot d\tau \circ\!\!-\!\!\bullet\ \frac{1}{j\omega} \cdot S(\omega) + \pi \cdot S(0) \cdot \delta(\omega)$ for $S(0) \neq 0$
13	Multiplication and convolution	$s_1(t) * s_2(t) = \int\limits_{-\infty}^{\infty} s_1(\tau) \cdot s_2(t-\tau) \cdot d\tau$	$s_1(t) * s_2(t) \circ\!\!-\!\!\bullet\ S_1(\omega) \cdot S_2(\omega)$ $s_1(t) \cdot s_2(t) \circ\!\!-\!\!\bullet\ \frac{1}{2\pi} \cdot S_1(\omega) * S_2(\omega)$
14	Parseval's theorem	$s_1(t), s_2(t)$ absolutely and squared integrable $s_1(t)$ real $\rightarrow$	$\int\limits_{-\infty}^{\infty} s_1(t) \cdot s_2(t) \cdot dt = \frac{1}{2\pi} \cdot \int\limits_{-\infty}^{\infty} S_1(-\omega) \cdot S_2(\omega) \cdot d\omega$ $\int\limits_{-\infty}^{\infty} s_1^2(t) \cdot dt = \frac{1}{2\pi} \cdot \int\limits_{-\infty}^{\infty} \lvert S_1(\omega)\rvert^2 \cdot d\omega$
15	Symmetry	$s(t) \circ\!\!-\!\!\bullet\ S(\omega)$ both known	then we find : $S\left(-\frac{t}{\gamma}\right) \circ\!\!-\!\!\bullet\ 2\pi\lvert\gamma\rvert \cdot s(\omega \cdot \gamma)$ γ=arbitrary real scale factor