Fundamentals of Electrical Engineering 3

Chapter 4

Locus curves

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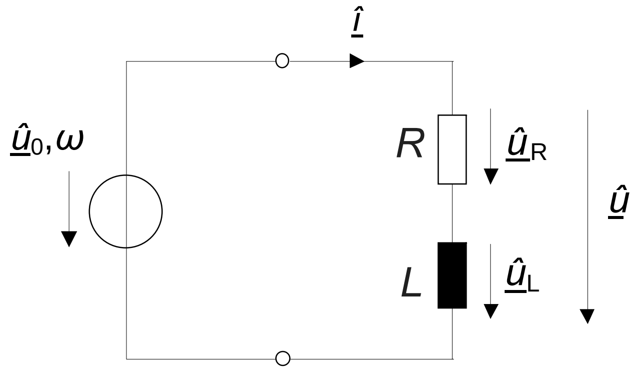
4 Locus curves

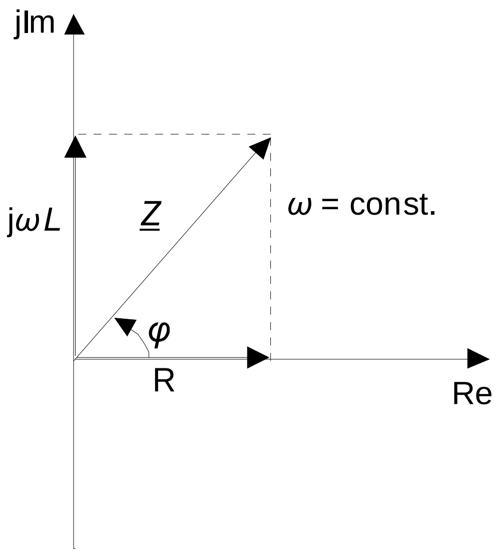
– A locus curve is one which connects end points of several/all complex pointers

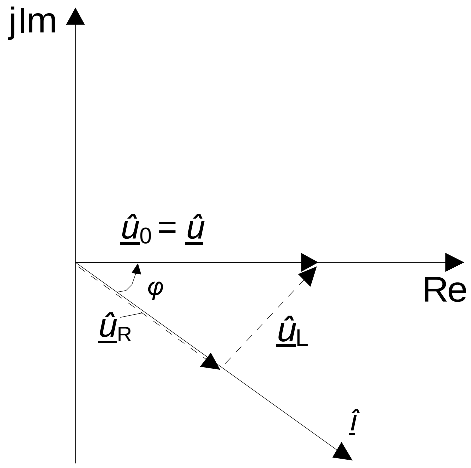
– A locus curve shows dependency of the frequency or another parameter

4 Locus curves



Example: A passive network





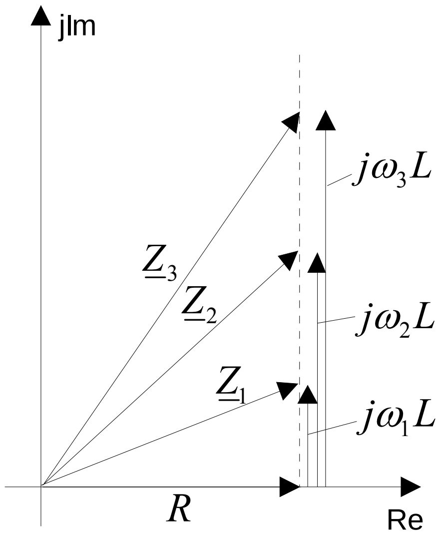
Relater pointer diagrams

It holds:

$$\varphi = \arctan\left(\frac{\omega L}{R}\right)$$

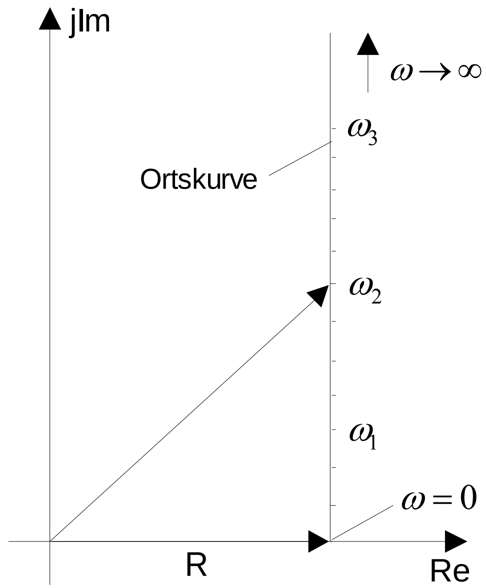
4 Locus curves

In some cases the impedance for several frequencies is of interest



$$\underline{Z}_1 = R + j\omega_1 L$$
$$\underline{Z}_2 = R + j\omega_2 L$$
$$\underline{Z}_3 = R + j\omega_3 L$$
$$\vdots \quad \vdots \quad \vdots$$

Impedance for 3 frequencies



Connection gives the locus curve

4 Locus curves

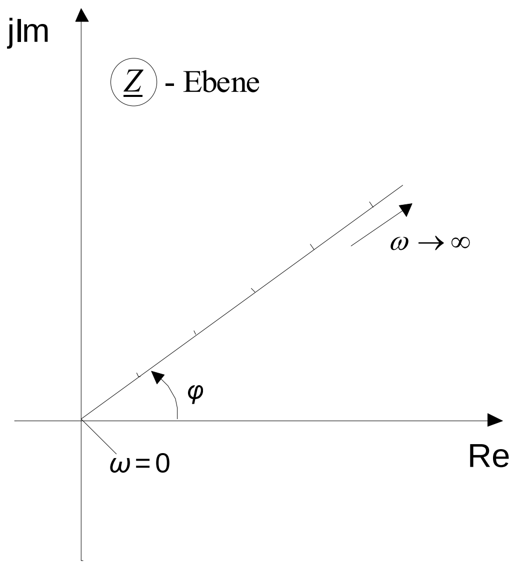
Comparison of Impedance and admittance pointers:

A straight line gives through the origin gives again another one through the origin

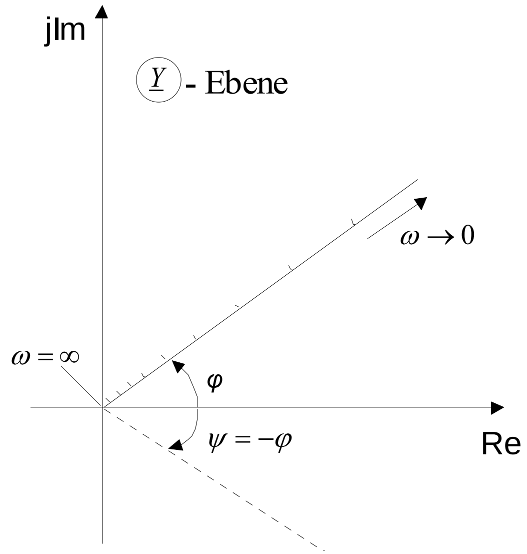
$$\underline{Z} = |\underline{Z}| e^{j\varphi} \qquad \underline{Z} = \frac{1}{\underline{Y}} \qquad \underline{Z}\underline{Y} = 1$$

$$\underline{Y} = |\underline{Y}| e^{j\psi}$$

$$\varphi = -\psi \qquad |\underline{Y}| = \frac{1}{|\underline{Z}|}$$



Locus curve of impedance

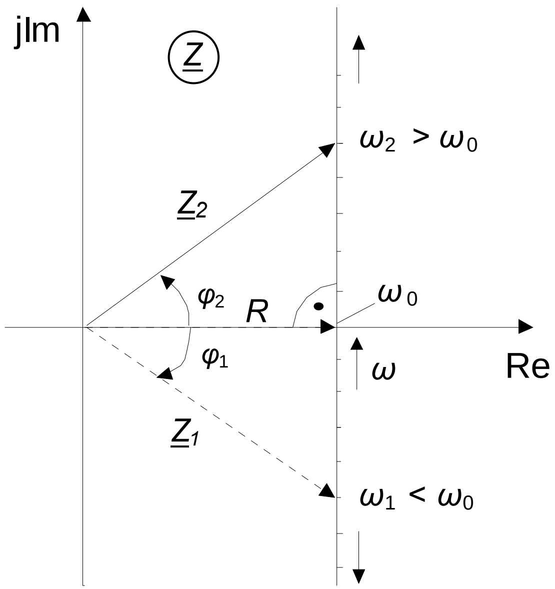


Locus curve of admittance

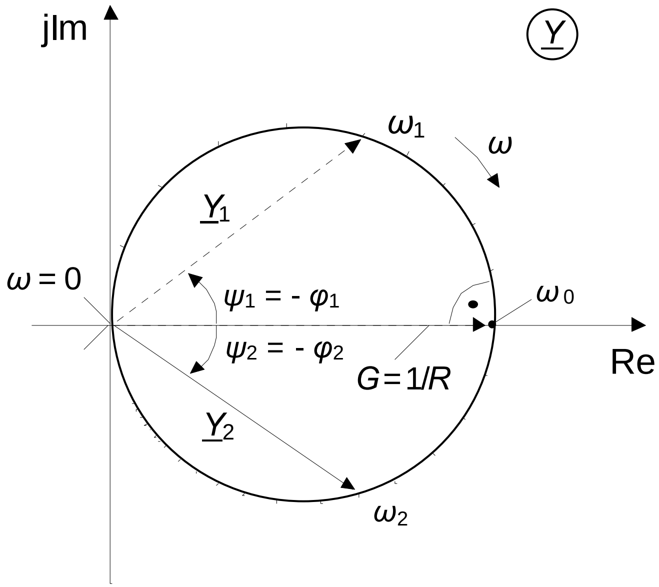
4 Locus curves

Each straight line not through the origin gives a circle through the origin.

Straight lines are always parallel to the imaginary axis and have positive real part for passive networks.



Locus curve of impedance

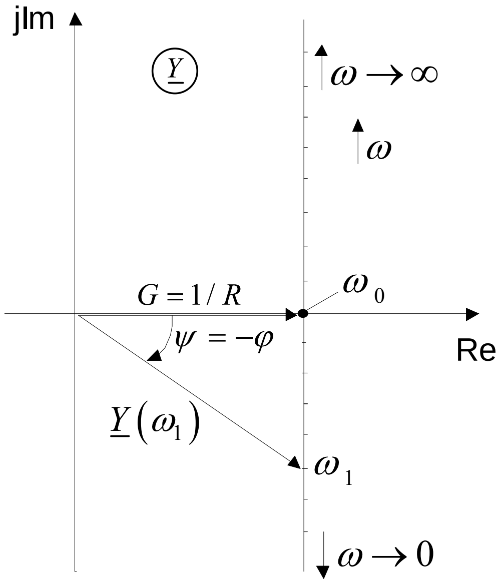


Locus curve of admittance and its diameter determination

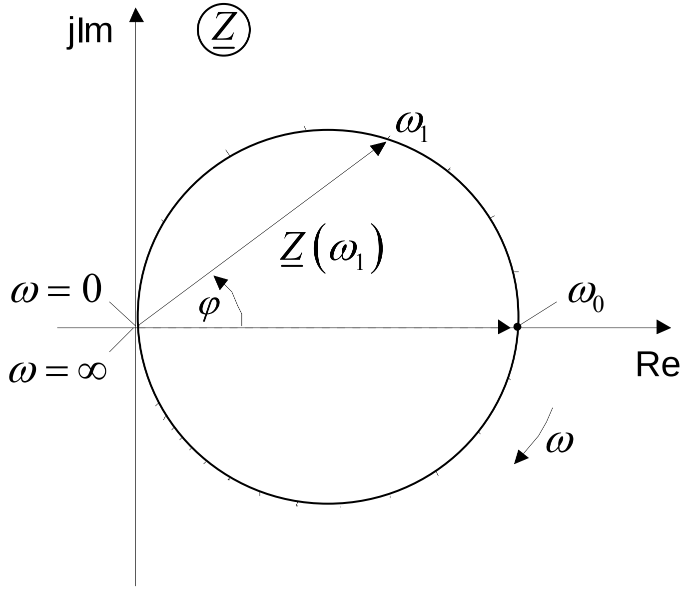
4 Locus curves

The transform is always accurate concerning angles in a small area

Right angles are kept , e.g. area at G = 1/R



Locus curve of admittanz
for parallel resonant circuit

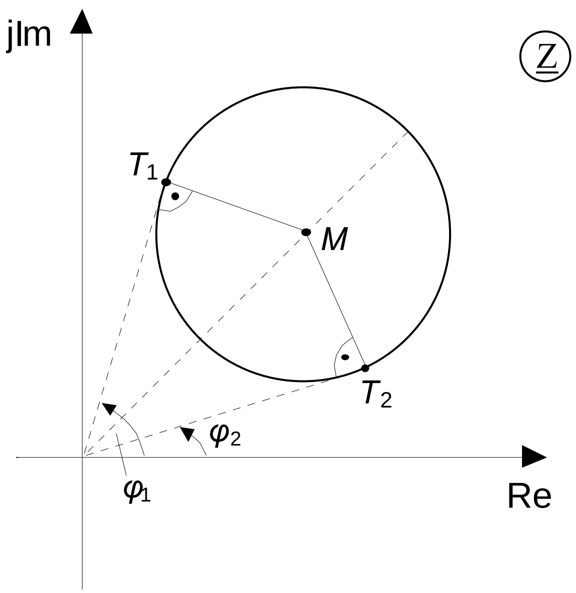


Locus curve of corresponding impedance

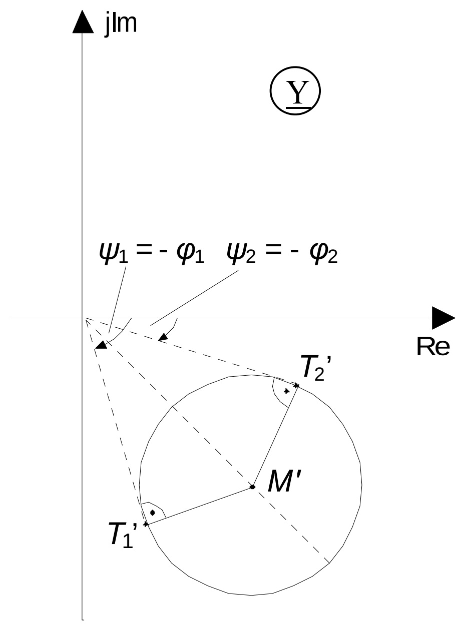
4 Locus Curves

Circles through the origin are transformed into straight lines parallel to the imaginary axis.

Circles not going through the origin are transformed in another one, again not going through the origin.



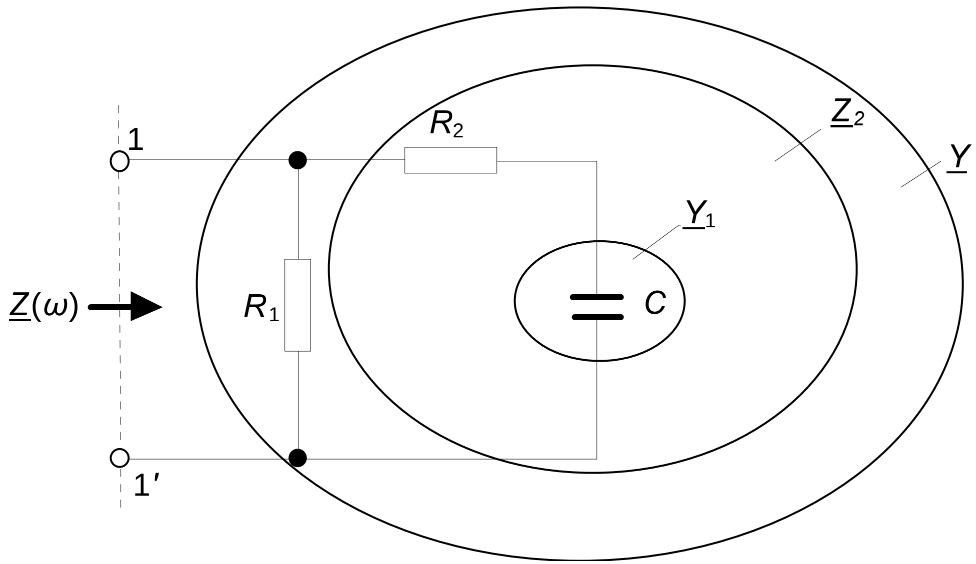
Locus curve of impedance



Locus curve of admittance

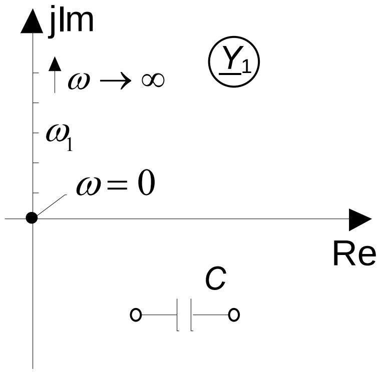
4 Locus curves

Example:

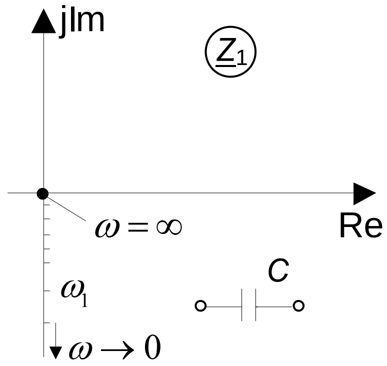


A passive network

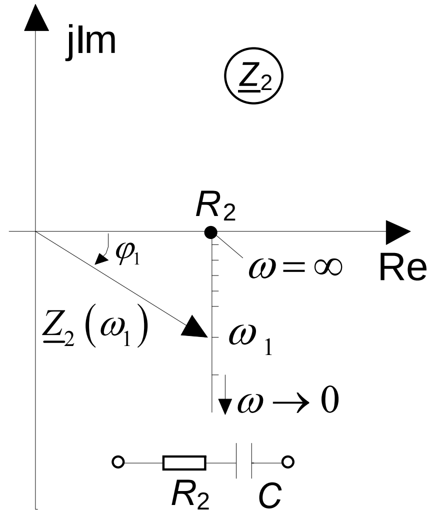
4 Locus curves



Locus curve of admittance for C

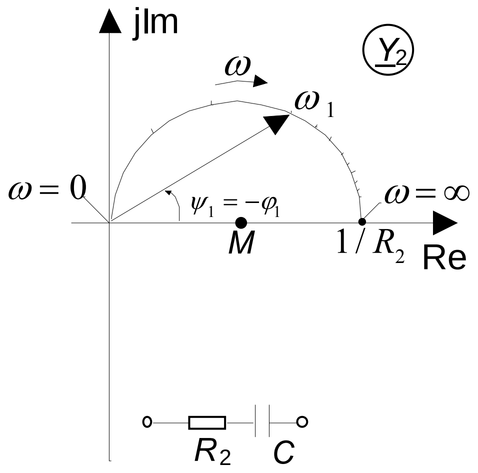


Corresponding locus curve of impedane for C

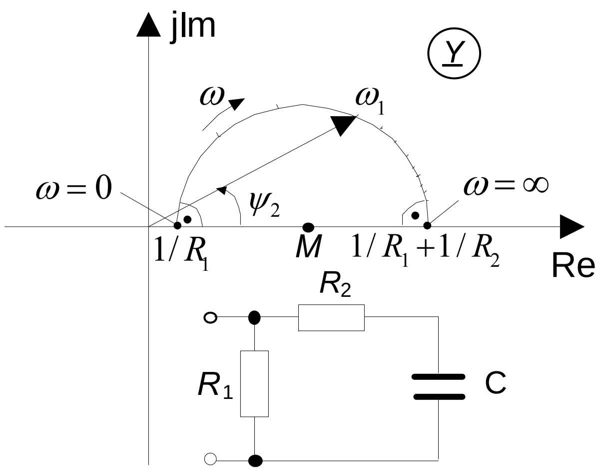


Impedance of serial connection

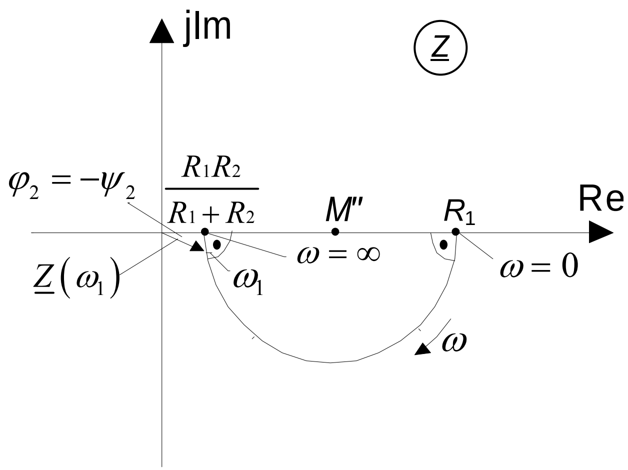
4 Locus curves



Locus curve of admittance of serial connection



Admittance of whole network



Locus curve of impedance of whole circuit

4 Locus curves

Disadvantages of locus curves:

- No or costly parametric description of values
- Costly construction of geometry

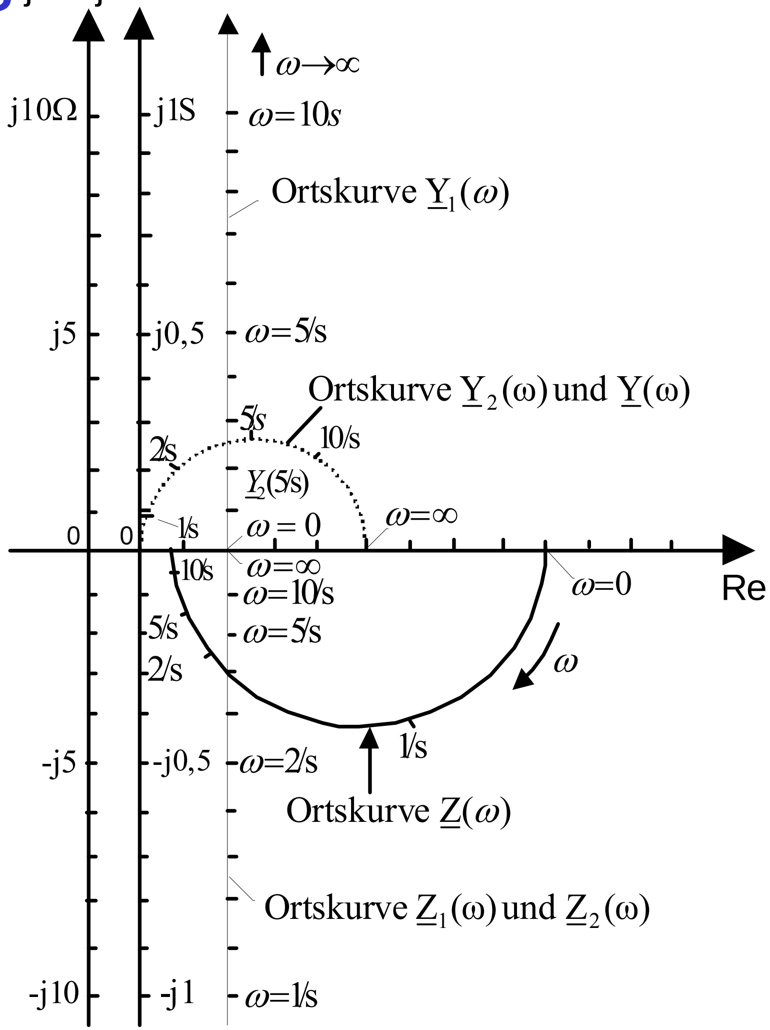
Example for $R_1 = 10\Omega \quad R_2 = 2\ \Omega \quad C = 0{,}1F$:

Drawing with e.g. 1 cm = 0,1 S for Y and 1 Ω = 1 cm for Z

4 Locus curves

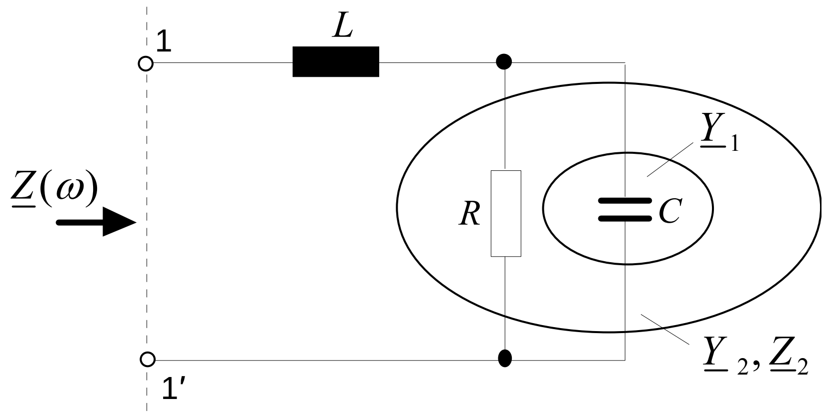
Scaling of locus curve gives the following drawing:

Grid is 1 cm wide

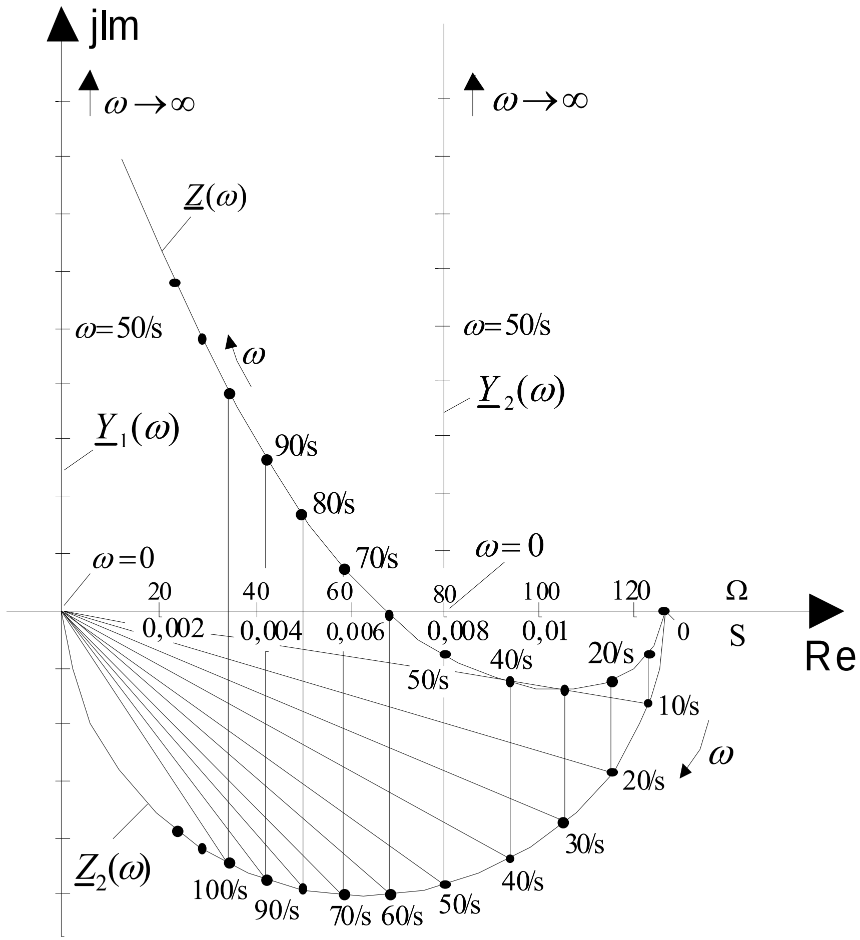


4 Locus curves

Second example with R =125 Ω , C = 100 µF und L = 0,85 H:



Grid caling: 10 Ω = 1 grid point for Z
1mS = 1 grid point for Y



Resulting locus curve

Grundlagen der Elektrotechnik 3

Chapter 5

Network theorems

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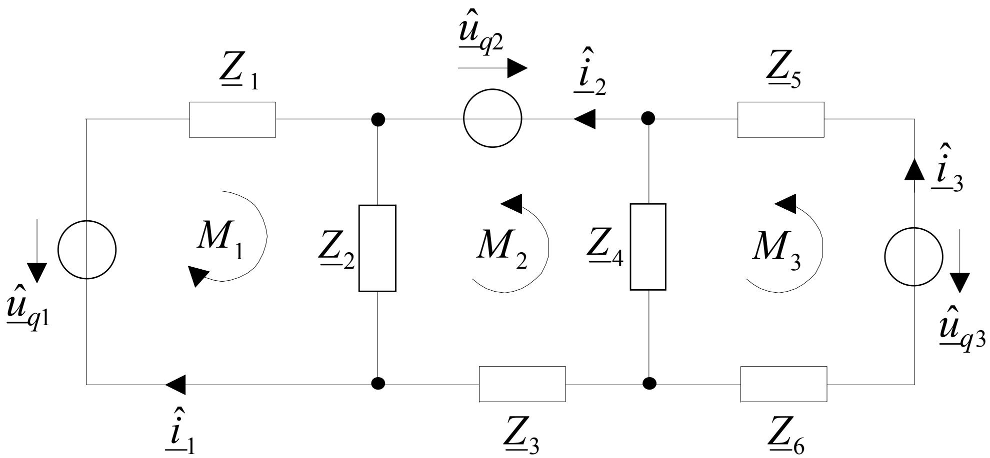
5.1 Der Überlagerungssatz

The superposition theorem is valid for all linear networks.

Example

A network contains 3 independent voltage sources.



The following loop equations are valid:

$$
\begin{aligned}
M_1:&\quad (\underline{Z}_1+\underline{Z}_2)\hat{\underline{i}}_1 + \underline{Z}_2\hat{\underline{i}}_2 &&= \hat{\underline{u}}_{q1}\\
M_2:&\quad \underline{Z}_2\hat{\underline{i}}_1 + (\underline{Z}_2+\underline{Z}_3+\underline{Z}_4)\hat{\underline{i}}_2 - \underline{Z}_4\hat{\underline{i}}_3 &&= \hat{\underline{u}}_{q2}\\
M_3:&\quad -\underline{Z}_4\hat{\underline{i}}_2 + (\underline{Z}_4+\underline{Z}_5+\underline{Z}_6)\hat{\underline{i}}_3 &&= \hat{\underline{u}}_{q3}
\end{aligned}
$$

5.1 Der Überlagerungssatz

A writing by matrices gives:

$$\underline{\overleftrightarrow{Z}}_{\mathrm{m}}\,\underline{\vec{\hat{i}}}_{\mathrm{m}} = \underline{\vec{\hat{u}}}_{\mathrm{qm}} \quad \text{mit} \quad \underline{\vec{\hat{u}}}_{\mathrm{qm}} = \begin{pmatrix} \underline{\hat{u}}_{\mathrm{q1}} \\ \underline{\hat{u}}_{\mathrm{q2}} \\ \underline{\hat{u}}_{\mathrm{q3}} \end{pmatrix}$$

According to the Cramer rule one obtains e.g.:

$$\underline{\hat{i}}_2 = \frac{-\underline{Z}_2(\underline{Z}_4+\underline{Z}_5+\underline{Z}_6)}{\det \underline{\overleftrightarrow{Z}}_{\mathrm{m}}}\underline{\hat{u}}_{\mathrm{q1}} + \frac{(\underline{Z}_1+\underline{Z}_2)\left(\underline{Z}_4+\underline{Z}_5+\underline{Z}_6\right)}{\det \underline{\overleftrightarrow{Z}}_{\mathrm{m}}}\underline{\hat{u}}_{\mathrm{q2}} + \frac{\underline{Z}_4(\underline{Z}_1+\underline{Z}_2)}{\det \underline{\overleftrightarrow{Z}}_{\mathrm{m}}}\underline{\hat{u}}_{\mathrm{q3}}$$

Interpretation:
This current is caused by superpositions of the effects of all 3 voltage sources.

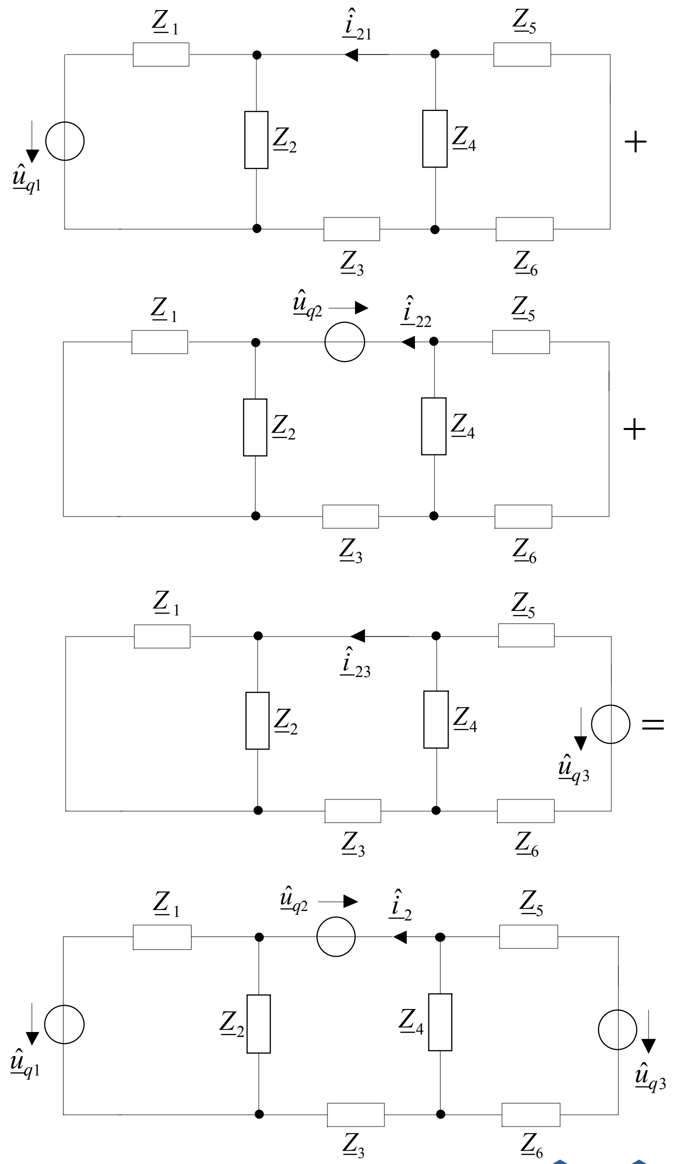
5.1 Der Überlagerungssatz

The total current contains current parts which are caused only by one of the 3 voltage sources.

$$\underline{\hat{i}}_2 = \underline{\hat{i}}_{21} + \underline{\hat{i}}_{22} + \underline{\hat{i}}_{23}$$

For the superposition principle the following rule hold:

If in a network q current or voltage sources are included all currents and all voltages can be determined by superposition of each of these sources.



5.1 Der Überlagerungssatz

Hint 1:
The theorem also holds for non sinusoidal sources. In this case complex pointers cannot be used.

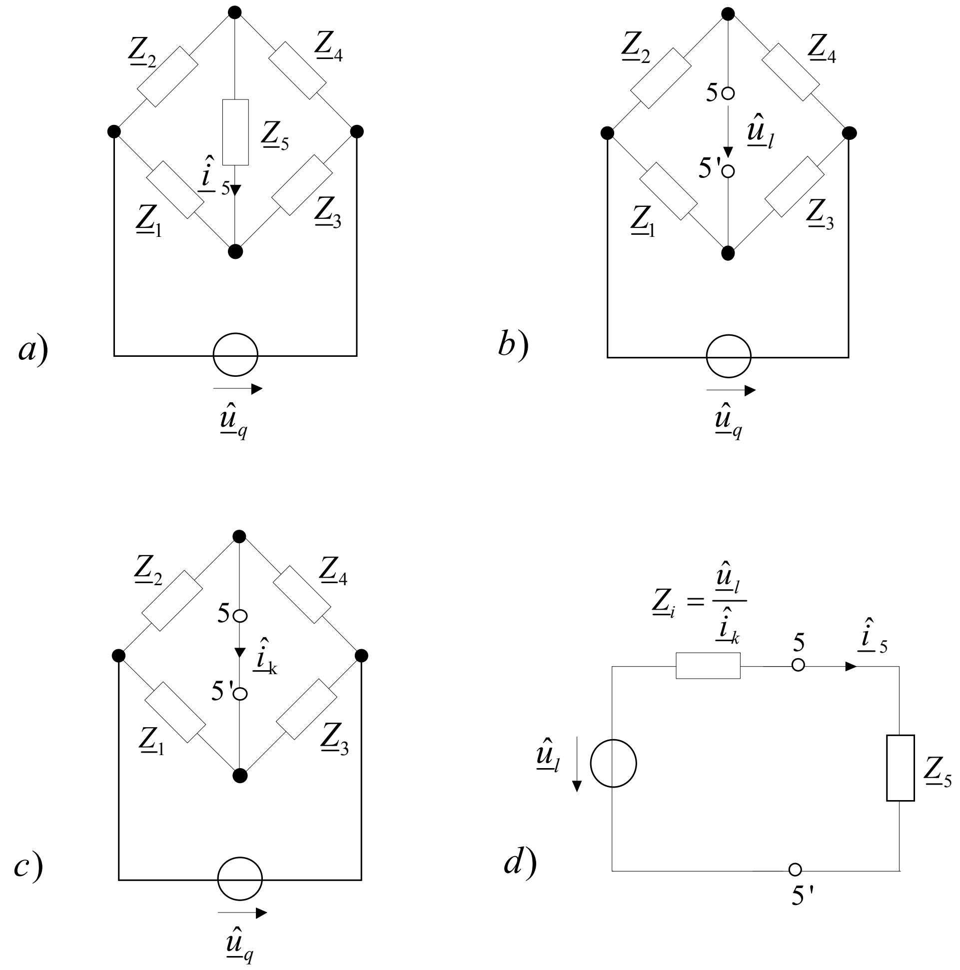
Hint 2:
If in a loop several voltage sources are included, at first these should be combined to one source before using the superposition.

The superposition gives the basis for determining currents and voltages in a network by the method of indepedent loop currents.

5.2 Der Satz zur Ersatzspannungsquelle

For complex networks sometimes it is only required to determine the effect of it to one load in a simplified procedure.

The following example shows this procedure:



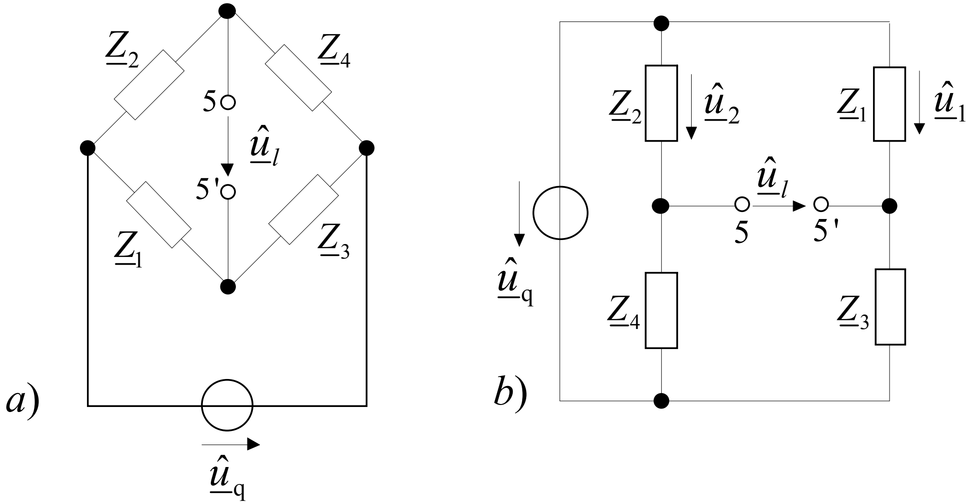
a) Network with attached load
b) Determination of open circuit voltage
(without load)
c) Determination of short circuit current
d) Resulting equivalent circuit

5.2 Der Satz zur Ersatzspannungsquelle

The calculation concerning b) of the example is performed using:

$$\underline{\hat{u}}_l = \underline{\hat{u}}_1 - \underline{\hat{u}}_2$$

$$= \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_3}\underline{\hat{u}}_q - \frac{\underline{Z}_2}{\underline{Z}_2 + \underline{Z}_4}\underline{\hat{u}}_q$$

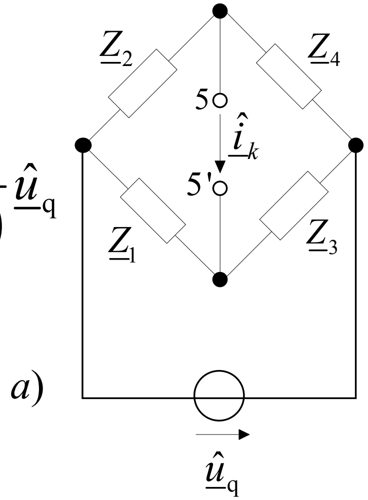


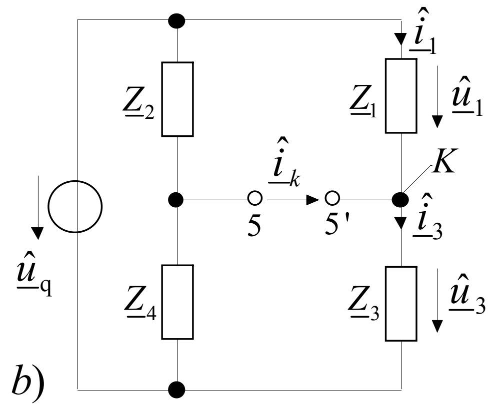
5.2 Der Satz zur Ersatzspannungsquelle

The calculation concerning c) of the example is performed using:

$$\underline{\hat{i}}_{\mathrm{k}} = \underline{\hat{i}}_3 - \underline{\hat{i}}_1$$

$$= \frac{\underline{Z}_4\underline{Z}_1 - \underline{Z}_2\underline{Z}_3}{\underline{Z}_1\underline{Z}_2\left(\underline{Z}_3+\underline{Z}_4\right)+\underline{Z}_3\underline{Z}_4\left(\underline{Z}_1+\underline{Z}_2\right)}\underline{\hat{u}}_{\mathrm{q}}$$





Other calculation steps are as follows:

5.2 Der Satz zur Ersatzspannungsquelle

First the voltages according to the voltage division rule are determined, then based on this the currents:

$$\underline{\hat{u}}_1 = \frac{\dfrac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}}{\dfrac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}+\dfrac{\underline{Z}_3\underline{Z}_4}{\underline{Z}_3+\underline{Z}_4}}\underline{\hat{u}}_\mathrm{q} \qquad \underline{\hat{u}}_3 = \frac{\dfrac{\underline{Z}_3\underline{Z}_4}{\underline{Z}_3+\underline{Z}_4}}{\dfrac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}+\dfrac{\underline{Z}_3\underline{Z}_4}{\underline{Z}_3+\underline{Z}_4}}\underline{\hat{u}}_\mathrm{q}$$

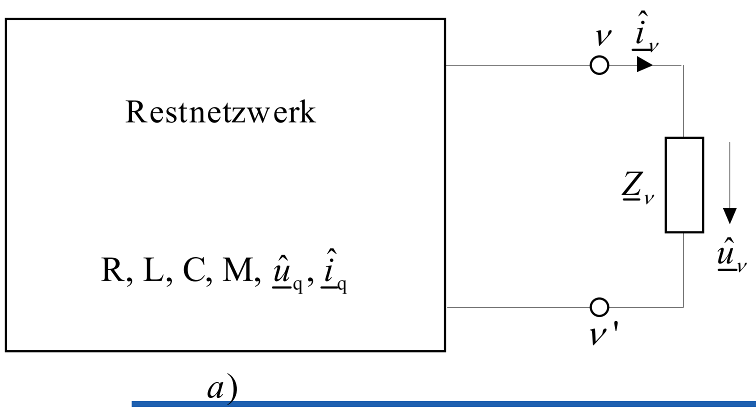
$$\underline{\hat{i}}_1 = \frac{\underline{u}_1}{\underline{Z}_1} = \frac{\dfrac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}}{\dfrac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}+\dfrac{\underline{Z}_3\underline{Z}_4}{\underline{Z}_3+\underline{Z}_4}}\underline{\hat{u}}_\mathrm{q} \qquad \underline{\hat{i}}_3 = \frac{\underline{u}_3}{\underline{Z}_3} = \frac{\dfrac{\underline{Z}_4}{\underline{Z}_3+\underline{Z}_4}}{\dfrac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}+\dfrac{\underline{Z}_3\underline{Z}_4}{\underline{Z}_3+\underline{Z}_4}}\underline{\hat{u}}_\mathrm{q}$$

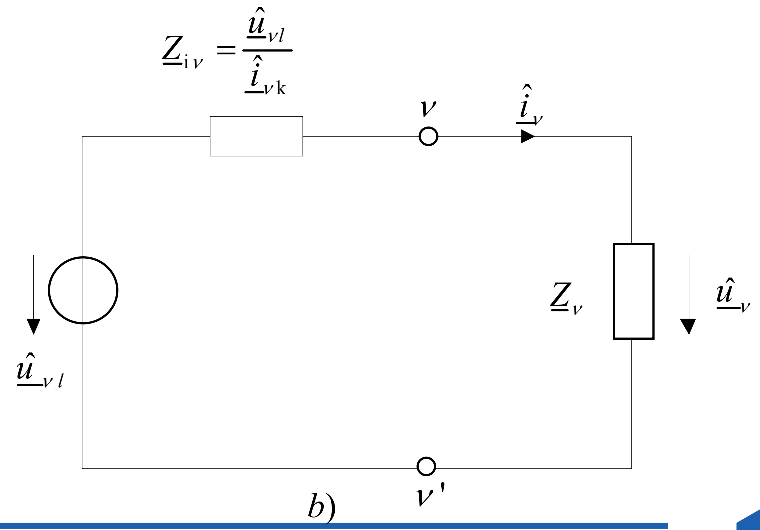
5.2 Der Satz zur Ersatzspannungsquelle

Theorem:
In a network out of passive components (with R,C,L,M components) which contains no controlled sources, current and voltages at a load can be determined by replacing the network with its equivalent circuit.

The open circuit voltage of it is identical to the open circuit voltage of the network without load.

The internal impedance of the equivalent circuit is determined by the open circuit voltage and the short circuit current (when replacing the load by a short circuit).





5.2 Der Satz zur Ersatzspannungsquelle

Alternative determination of the internal impedance:
- Replacement of all current sources by open circuits
- Replacement of all voltage sources by short circuits
- Determination of the output impedance of the remaining network by conventional network analysis

5.3 Der Satz zur Ersatzstromquelle

An alternative determination uses equivalent networks with one current source:

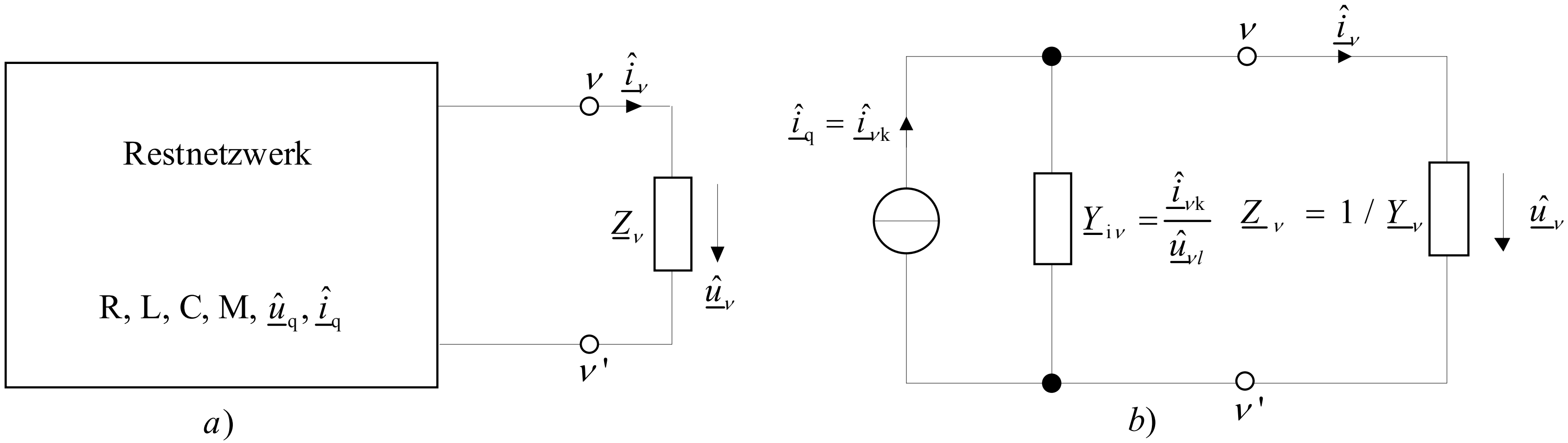
Theorem:
In a passive network without controlled sources, voltage and current at a load can be determined by replacing the network with an equivalent current source.

The internal admittance results from the open circuit voltage and the short circuit current

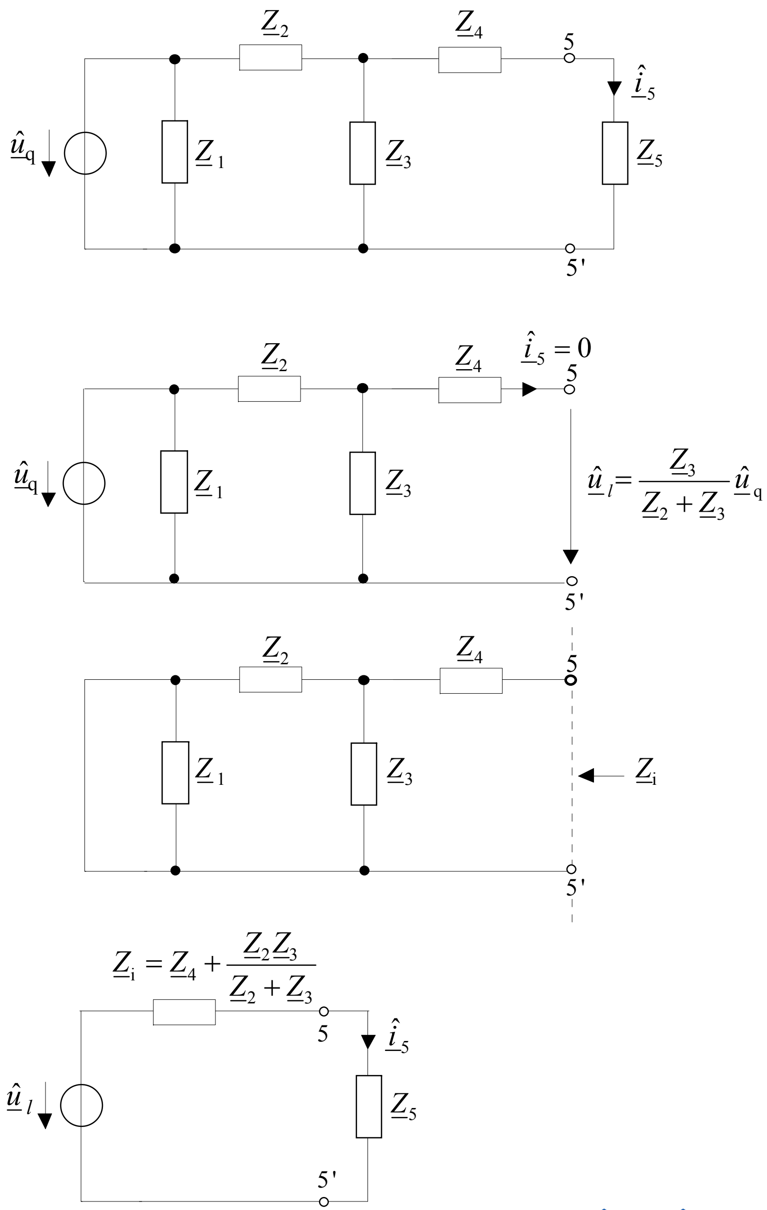
Alternative determination of the internal admittance:
- Replacement of all current sources by open circuits
- Replacement of all voltage sources by short circuits
- Determination of the output admittance of the remaining network by conventional network analysis

5.2 Der Satz zur Ersatzspannungsquelle

Here the equivalent circuit is shown:



5.3 Weitere Beispiele



Example 1

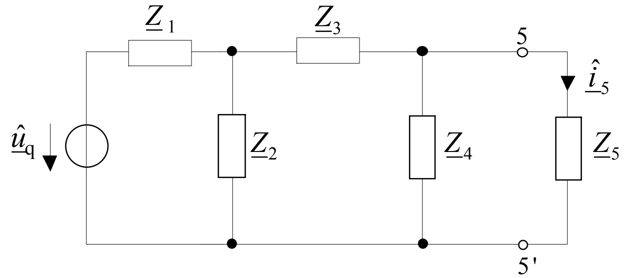
The internal impedance is here determined by network analysis of the remaining network.

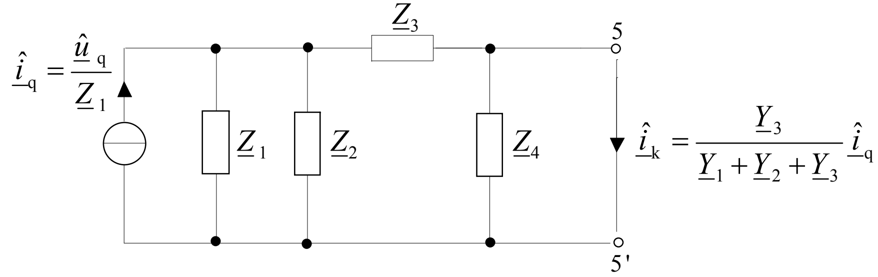
$$\hat{\underline{i}}_5 = \frac{\hat{\underline{u}}_l}{\underline{Z}_i + \underline{Z}_5} = \frac{\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3}\hat{\underline{u}}_q \frac{1}{\underline{Z}_4 + \frac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} + \underline{Z}_5}$$

$$= \frac{\underline{Z}_3}{\underline{Z}_2\left(\underline{Z}_3 + \underline{Z}_4 + \underline{Z}_5\right) + \underline{Z}_3\left(\underline{Z}_4 + \underline{Z}_5\right)}\hat{\underline{u}}_q$$

5.3 Weitere Beispiele

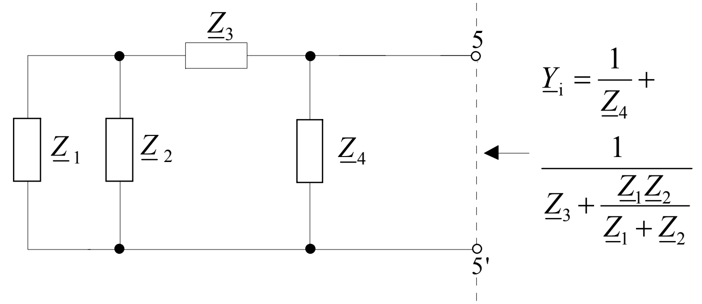
Example 2





It holds:

$$\underline{\hat{i}}_5 = \frac{\underline{Y}_5}{\underline{Y}_i + \underline{Y}_5}\,\underline{\hat{i}}_k$$



After some calculations it results:

$$\underline{\hat{i}}_5 = \frac{\underline{Z}_2\underline{Z}_4}{\left(\underline{Z}_4 + \underline{Z}_5\right)\left(\underline{Z}_1\underline{Z}_2 + \underline{Z}_1\underline{Z}_3 + \underline{Z}_2\underline{Z}_3\right) + \underline{Z}_4\underline{Z}_5\left(\underline{Z}_1 + \underline{Z}_2\right)}\,\underline{\hat{u}}_q$$