Exercises

of the lecture

Fundamentals of Electrical Engineering 3

Department

Communication Systems

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and

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Exercise 1:

1. Sketch the following functions

a) $s(t) = A \cdot \text{rect}\left(\frac{t - t_0}{T_0}\right)$

b) $\varepsilon(t - T_0) \cdot \sin(\omega_0 \cdot t + \varphi_0)$; $\varphi_0 = \frac{2\pi}{3}$; $\mathrm{T}_0 = \frac{2\pi}{\omega_0}$

c) $\Lambda\left(\frac{t}{T} - 2\right) + \Lambda\left(\frac{t}{T} + 2\right)$

d) $\text{rect}\left(\frac{t}{T} + \frac{1}{2}\right) \cdot \mathrm{r}\left(-\frac{t}{T}\right)$

e) $\text{rect}\left(\frac{t}{2T_0}\right) \cdot \sin(\omega_0 t)$

f) $\Lambda\left(\frac{t}{T}\right) \cdot \Lambda\left(-\frac{t}{T}\right)$

g) $\varepsilon\left(-t + \frac{T}{2}\right) \cdot \text{rect}\left(\frac{t}{2T}\right)$

h) $\Lambda\left(\frac{2t}{T}\right) + \text{rect}\left(\frac{\mathrm{t}}{2\mathrm{T}}\right)$

i) $\sum_{n=-2}^{+2} \delta(t - nT) \cdot \Lambda\left(\frac{t}{2T}\right)$

j) $\delta(2t + T) - 2\delta(-3t - 2T)$; Note : $\delta(at) = \frac{1}{|a|}\delta(t)$

2. Determine the formula describing the following signal

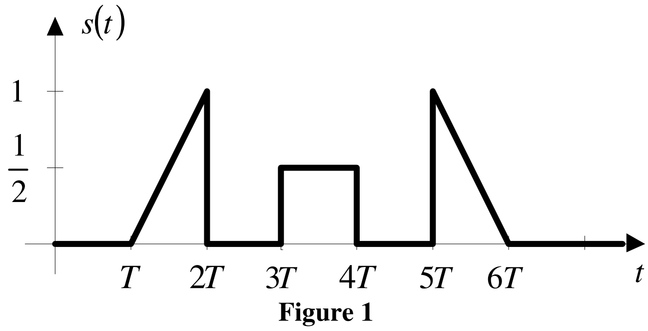


Figure 1

Exercise 2:

A signal $s(t)=\sum_i A\cdot\Lambda\left(\frac{t-iT_0}{\frac{T_0}{2}}\right)$ is given with $A=1\text{V}$ and $T_0=1\text{ms}$.

a) Sketch the function $s(t)$
b) Determine the Fourier coefficients a_n and b_n of the Fourier series of $s(t)$ for all $|n|<6$

Exercise 3:

The signal $s_1(t)$ is given $(T=1\text{ms})$.

$$s_1(t)=4+2\cdot\sin\left(\frac{2\pi t}{T}\right)+3\cdot\sin\left(\frac{8\pi t}{T}\right)+4\cdot\cos\left(\frac{8\pi t}{T}\right)$$

a) Write $s_1(t)$ in all other forms (trigonometric, polar, exponential resp.)
b) Which harmonics are present?
c) Sketch the two sided magnitude-/ phase spectrum $\left(\text{in the form of plots over } f=\frac{1}{T}\right)$
d) Determine the RMS value

Exercise 4:

Given is a periodic voltage signal with the period T_0.

$$\underline{c}_n=\begin{cases}1\text{V}\cdot e^{-j\frac{\pi}{4}} & \text{for } n=-1\\ 1\text{V}\cdot e^{+j\frac{\pi}{4}} & \text{for } n=+1\\ 0 & \text{else}\end{cases}$$

a) Determine the Fourier coefficients a_n and b_n
b) Specify the expression for $s(t)$ including its dimension in trigonometric and polar form
c) Specify the distortion factor

Exercise 5:

Determine the Fourier coefficients c_n, a_n and b_n for the following signals

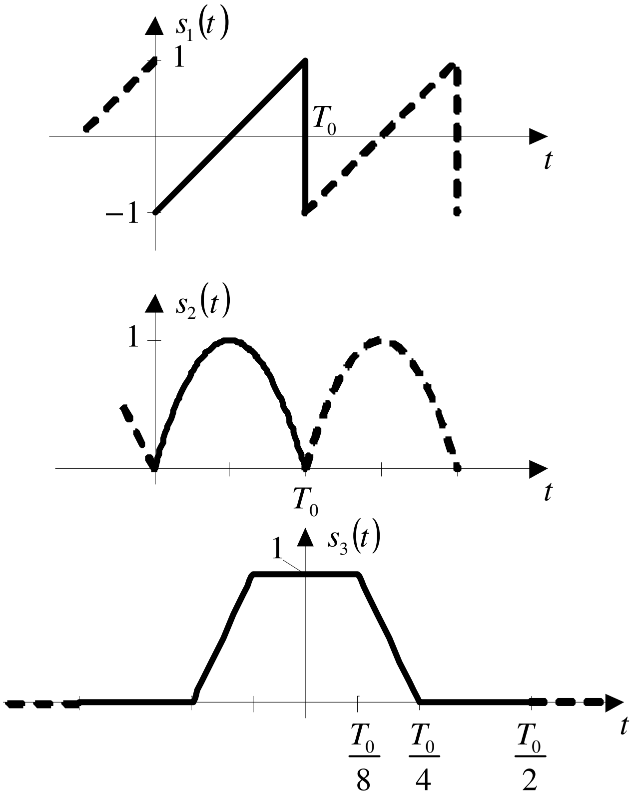


Figure 5

Note: $s_3(t)$ is an even signal of the width $\frac{T_0}{2}$!

Exercise 6:

An even signal $s(t) = \sum_i 1\text{V}\ \Lambda\left(\frac{t - i \cdot T_0}{\frac{T_0}{2}}\right)$ with $T_0 = 1\text{ms}$ is given

a) Determine the Fourier coefficients a_n and b_n

b) Write down the Fourier transform $S_F(\omega)$ of a single pulse $\Lambda\left(\frac{t}{\frac{T_0}{2}}\right)$ and sketch this function

c) Show that for $f = 1\text{kHz}$ and 3kHz the relation $\underline{c}_n = \frac{S_F(2\pi f)}{T_0}$ applies

Exercise 7:

In Figure 7a a periodic voltage signal $u_1(t)$ is given.

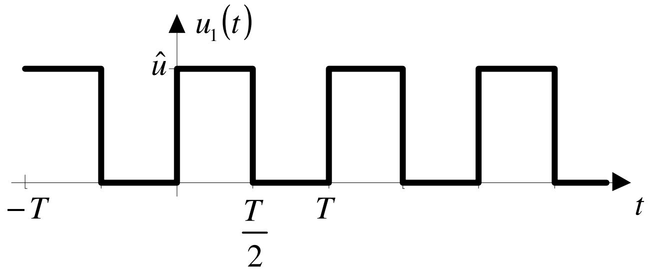


Figure 7a

a) Determine the Fourier coefficients c_n and sketch the amplitude spectrum
b) Determine the Fourier coefficients c_n for $u_2(t)$, if $u_1(t)$ is input to the following circuits. Determine the corresponding amplitude spectra

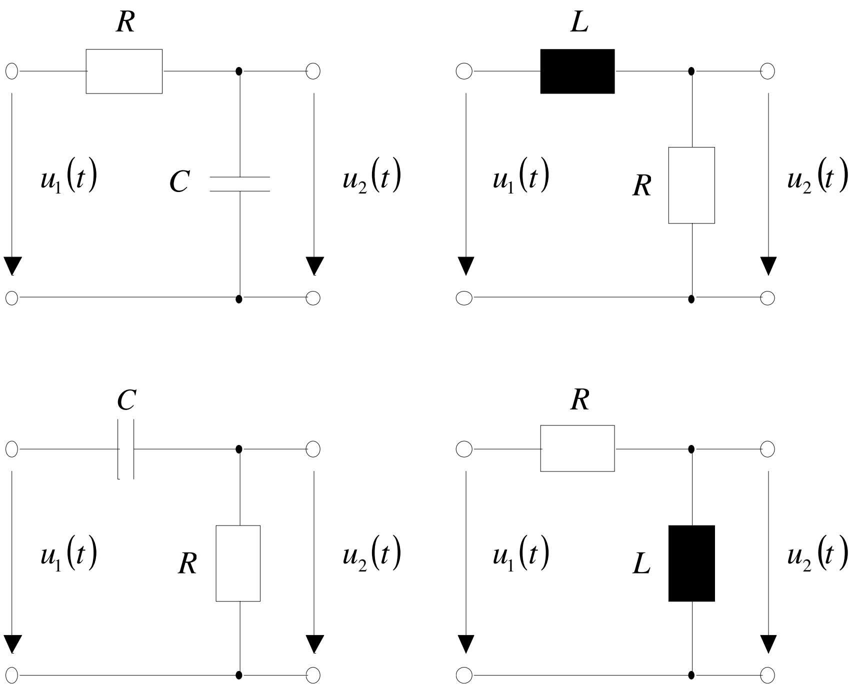
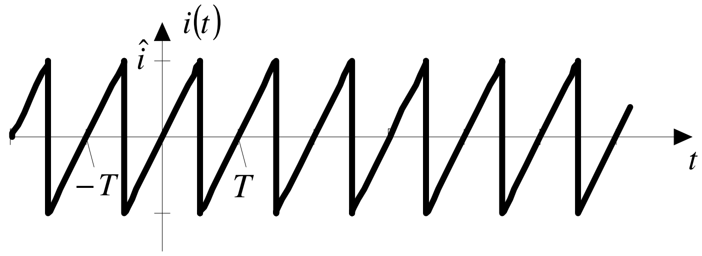


Figure 7b

Exercise 8:

The depicted current signal $i(t)$ of Fig. 8 is input to the also represented circuit.



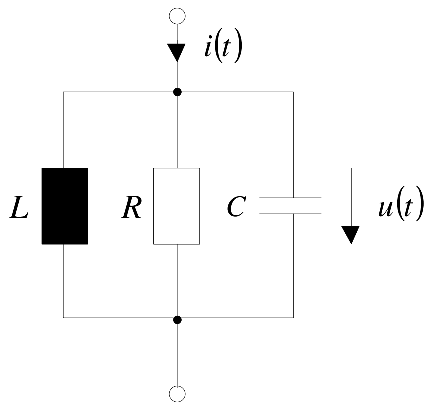


Figure 8

It holds: $L = \dfrac{1}{4\omega^2 C} = \dfrac{T^2}{16\pi^2 C}$

a) Determine the Fourier coefficients c_n of $i(t)$ and of $u(t)$

b) Sketch the amplitude spectra of $i(t)$ and of $u(t)$ for quality factor values of the resonant circuit of $Q = 1$ and $Q = 10$, respectively.

c) Determine for $Q = 1$ and $Q = 10$

    i. the RMS values I_{rms} and U_{rms}

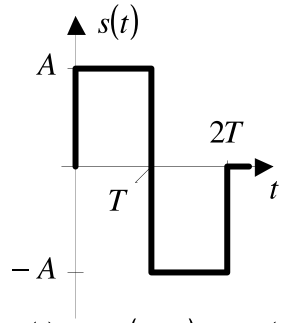
    ii. the oscillation content and the basic oscillation amount of $i(t)$ and of $u(t)$

    iii. the distortion factor of $i(t)$ and of $u(t)$

Exercise 9:

Determine the Fourier transform $S_F(\omega)$ of the following signals

a)



b) $s(t) = \sin(\omega_0 t) + \cos(\omega_0 t)$

Exercise 10:

Sketch and find $S_F(\omega)$ for:

$$s_1(t) = \exp\left(-2\frac{|t|}{T}\right)$$

$$s_2(t) = \text{rect}\left(\frac{t}{T} - \frac{1}{2}\right) - \Lambda\left(\frac{t - \frac{T}{2}}{\frac{T}{2}}\right)$$

Exercise 11:

Given is the function

$$f(t) = t \cdot \exp(-2t^2) + \int_{-\infty}^{\infty} \text{rect}\left(\frac{t-\tau}{T}\right) \cdot \text{rect}\left(\frac{\tau - T}{T}\right) \mathrm{d}\tau + \sin^2(\omega_0 t) \cdot \cos(\omega_0 t)$$

a) Determine the Fourier-transform $F(\omega)$ of the function $f(t)$ by using the correspondence tables.

b) Determine the magnitude $|G(\omega)|$ of the Fourier-transform of the function
$g(t) = \text{rect}\left(\frac{t}{2T}\right) * \text{rect}\left(\frac{t}{2T}\right)$.

c) Simplify the trigonometric expression $h(t) = 4\sin^2(\omega_0 t)\cos(\omega_0 t) + \cos(3\omega_0 t)$.

d) Determine the real part $S_R(\omega)$ of the Fourier-transform of the function
$s(t) = \cos(\omega_0 t) \cdot t \cdot \exp(-2t^2)$.

Exercise 12:

Given is the RC-low-pass with the impulse response

$$h(t) = \omega_0 \cdot \exp(-\omega_0 t) \cdot \varepsilon(t)$$

a) Determine the transfer function $H(\omega)$.
b) Determine the output signal $g(t)$ of the RC-low-pass for $s(t) = \varepsilon(t)$.
Hint: $G(\omega) = H(\omega) \cdot E(\omega)$, where $E(\omega)$ is the Fourier-transform of $\varepsilon(t)$.
c) Determine the output signal $g(t)$ of the RC-low-pass for $s(t) = \cos(\omega_0 t)$. Use the transfer function in general form $H(\omega) = |H(\omega)| \exp(j\varphi(\omega))$.

Exercise 13:

Determine the impulse response of a linear, time-invariant transmission system with the sketched transfer function

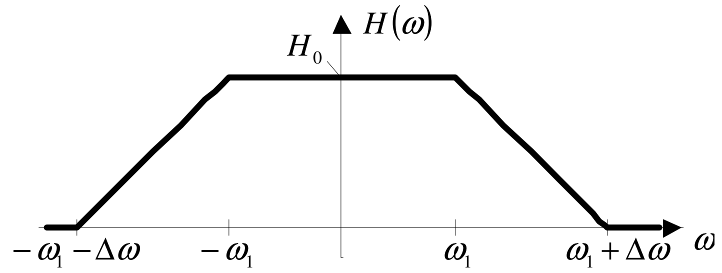
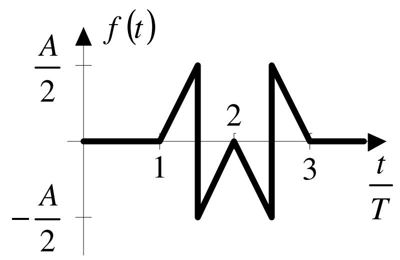


Figure 13

Hint:
Represent the sketched transfer function as a convolution of two rect-functions and determine the impulse response by means of the standard table of Fourier-transforms.

Exercise 14:

Determine the Laplace-transform $F(p)$ of
a) the function $f(t) = -t \cdot \sin(\omega_0 t)$
b) the below depicted function $f(t)$



Exercise 15:

Determine the function $f(t)$ by means of inverse Laplace-transform of the following functions.

a) $F(p)=\dfrac{p-1}{p\,\left(p^2+a^2\right)}$

b) $F(p)=\dfrac{1}{(p-a-\mathrm{j}b)(p-a+\mathrm{j}b)}$

Exercise 16:

a) Determine $h(t)$ for the RC-low-pass. Note that this low-pass responds to $s(t)=\varepsilon(t)$ with

$$u_C(t)=g(t)=\left(1-\mathrm{e}^{-\frac{t}{RC}}\right)\quad\forall\ t\geq 0$$

b) Describe graphically the responds of the RC-low-pass to $\operatorname{rect}\left(\dfrac{t}{T}-0.5\right)$ with $\tau=RC$, and $\tau\in\{0.2,1.0,5.0\}\cdot T$

<u>Note:</u> $\mathrm{e}^{-1}=0.63;\ \mathrm{e}^{-5}=0.007;\mathrm{e}^{-0.2}=0.82$

c) Do the same for the RC-high-pass. <u>Note:</u> $u_R(t)+u_C(t)=s(t)$

d) Determine $h(t)$ by means of Laplace-transform of $u_R(t)+u_C(t)=s(t)$

e) Determine $H(\omega)$ by means of network analysis and using impedances

f) Determine $H(\omega)$ for the following two networks (still using network analysis)

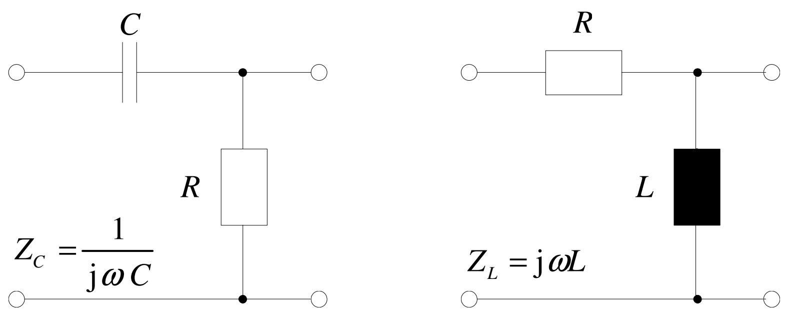


Figure 16

Exercise 17:

Given is the following schematic:

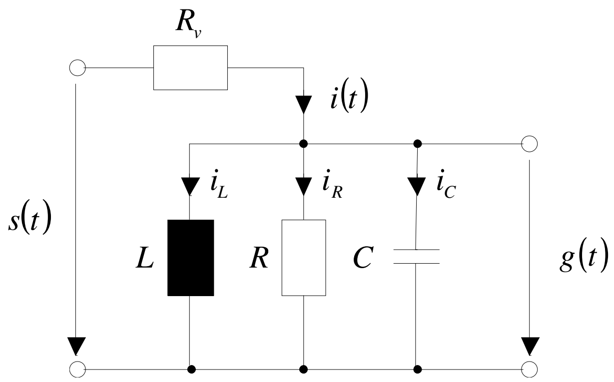


Figure 17

a) Determine the differential equation for this system
b) Determine the transfer function $H(\omega)$ and the system function $H_L(p)$

Exercise 18:

Given is the circuit of Fig. 18.

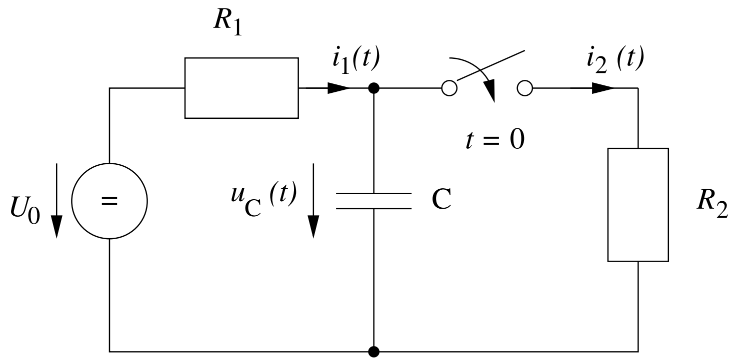


Figure 18

At $t = 0$ the switch is closed. Determine the voltage $u_C(t)$ and the currents $i_1(t)$ and $i_2(t)$ for $0 < t < \infty$.

Exercise 19:

Given is the circuit of Fig. 19.

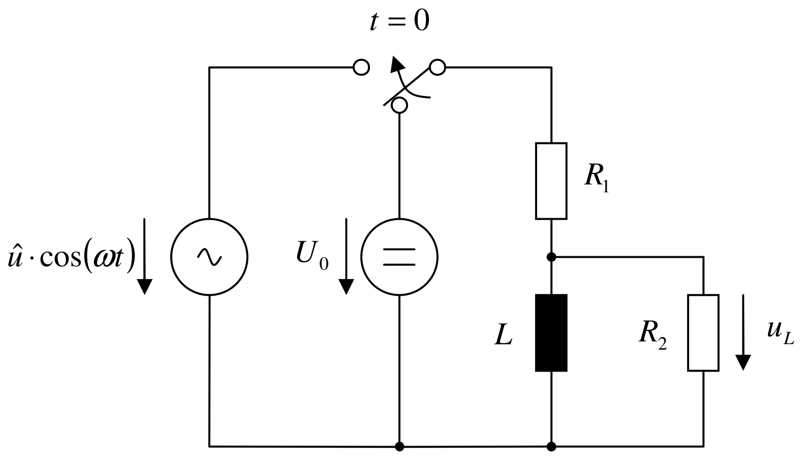


Figure 19

At $t=0$ the switch is closed. Determine the voltage signal $u_L(t)$.

Exercise 20:

Given is the circuit of Figure 20.

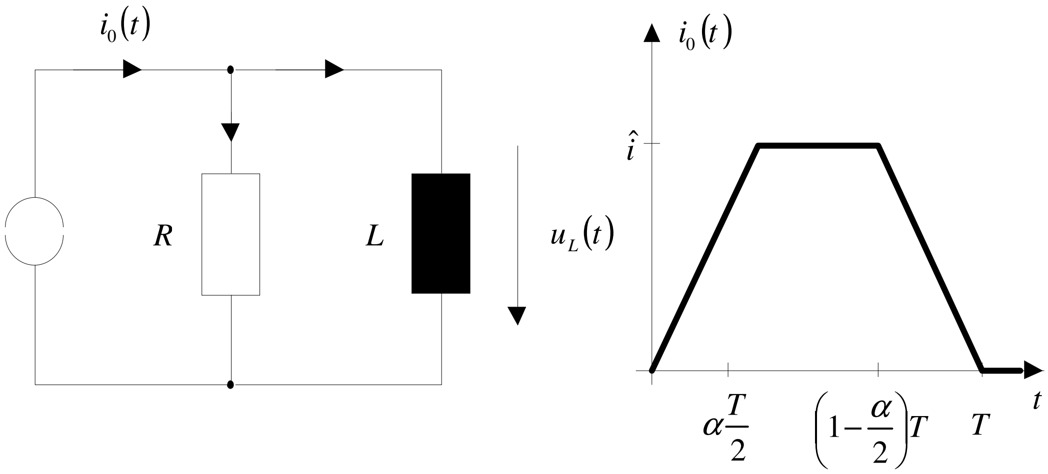


Figure 20

The time depended source current $i_0(t)$ is depicted in Figure 20 also. Determine

a) the voltage $u_L(t)$ of Figure 20
b) the voltage $u_L(t)$ for $R\to\infty$

by means of Laplace transformation.

Exercise 21:

Given is the circuit of Fig 21.

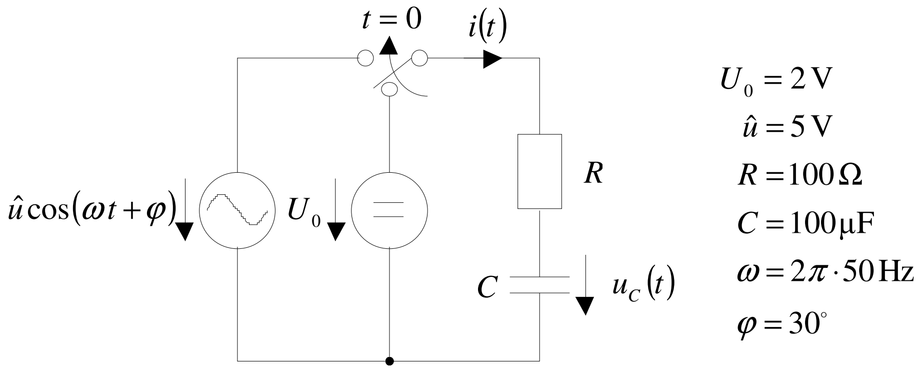
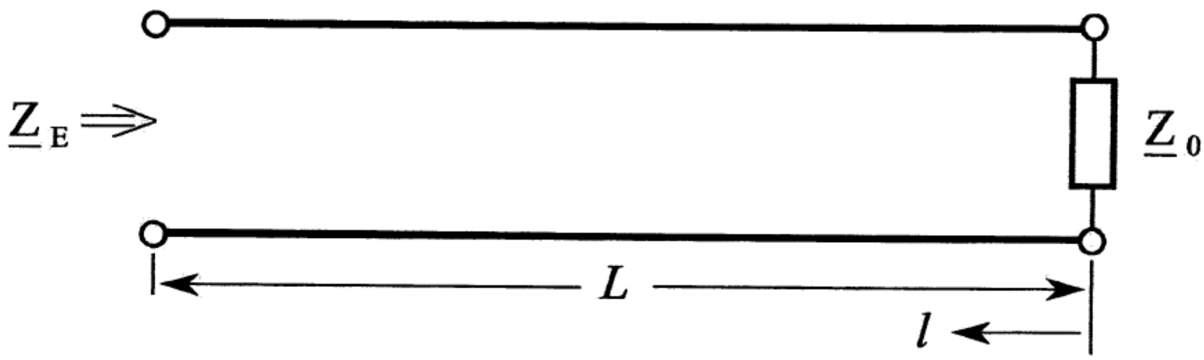


Figure 21

At $t = 0$ the switch is switched. Determine the voltage signal $u_C(t)$ and the current $i(t)$ by means of Laplace-transformation. Sketch $u_C(t)$ and $i(t)$ as a function of t.

Exercise 22:

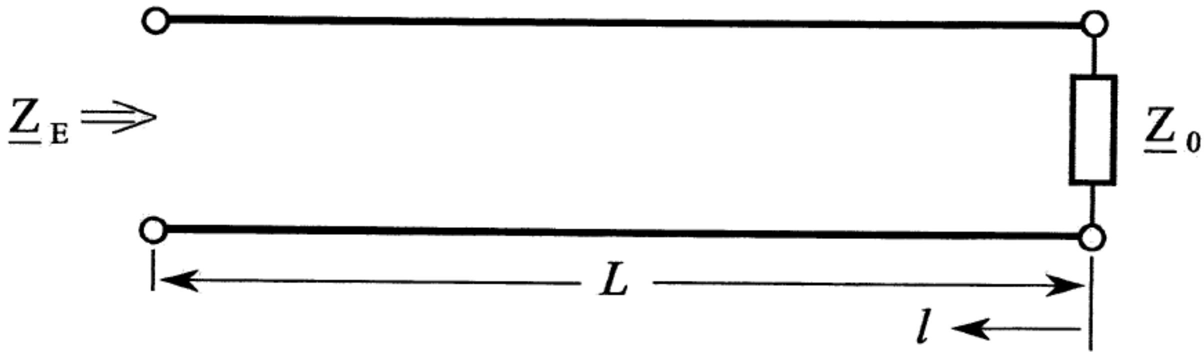
The following homogeneous low lossy line of length L with the complex wave impedance $\underline{Z}_L$ and the propagation constant $\underline{\gamma} = \alpha + j\beta$ is given. At the position $l = 0$ an arbitrary load Z_0 is applied to the two port.



Determine for the angular frequency $\omega = 2\pi f$ the input impedance $\underline{Z}_E$ of the low lossy line to which the arbitrary load $\underline{Z}_0$ is applied.

Exercise 23:

A homogeneous lossless line of the length L is given. The distributed inductance L' and the distributed capacitance C' are both given. For the calculation of the two-port the line is driven with a sinusoidal voltage with constant wavelength λ.



a) Determine the equations for the lossless line including the characteristic values based on the equations for the low lossy line.

b) Determine the input impedance $\underline{Z}_E$ of a lossless line to which the load $\underline{Z}_0$ is applied.

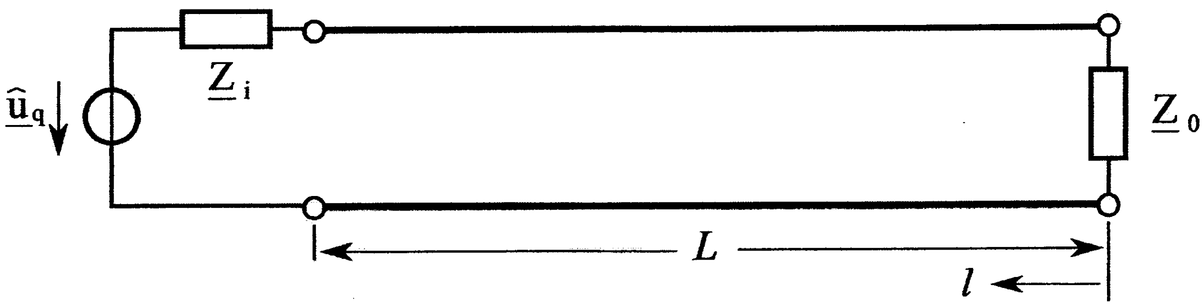
c) Show that for a particular line length $L = 0{,}25\ \lambda$ a resistance value $Z_0 = R_1$ can be transformed to a resistance value $\underline{Z}_E = R_2$ and determine the relation between both values.

d) Determine and sketch the current and voltage behaviour of the line for the following cases.

1. $Z_0 = 0$ (short circuited line)
2. $Z_0 = \infty$ (open loop)
3. $Z_0 = Z_L$ (matching)

Exercise 24:

A homogeneous lossless line of the length *L,* the distributed inductance *L'* and the distributed capacitance *C'* are given. For the calculation of the two-port the line is driven with an alternating voltage source. At the position $l = 0$ a load Z_0 is applied to the two port where the active power $P_0 = 180mW$ is converted to heat.



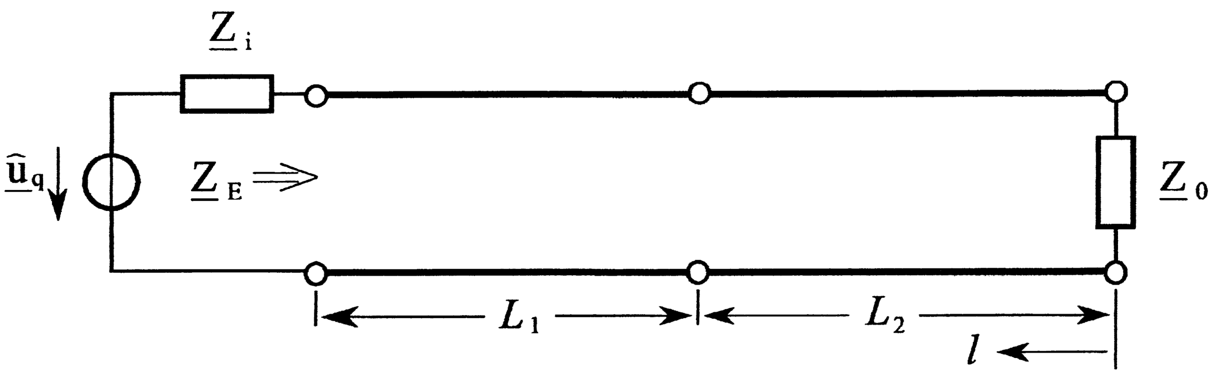
The following parameters are given:

$$\underline{Z}_0 = 100\Omega \; , \; \underline{Z}_i = 100\Omega \; , \; L = 1{,}25m \; , \; C' = 50\frac{pF}{m} \; , \; L' = 2\frac{\mu H}{m} \text{ and } f = 10MHz$$

Determine the peak amplitude voltage $\hat{\underline{u}}_q$ of the alternating voltage source which is connected to the line.

Exercise 25:

A homogeneous lossless line of the length L_1, and L_2 is given. The transmission line is connected to an alternating voltage source. At l = 0 a load $\underline{Z}_0$ is applied to the two port. The line L_1 has the wave impedance $\underline{Z}_{L1}$ and L_2 has the wave impedance $\underline{Z}_{L2}$.



The following parameters are given:

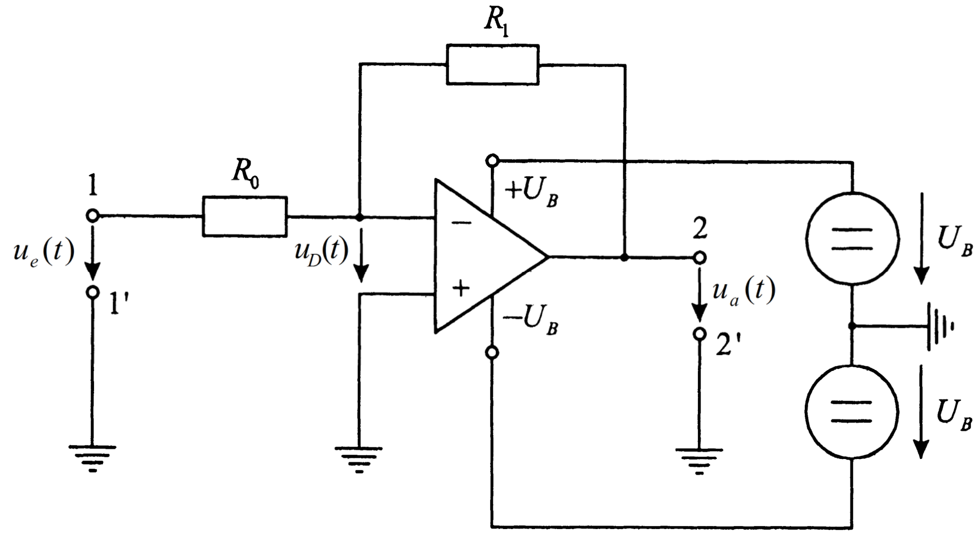
$\underline{Z}_0 = 80\Omega$, $\underline{Z}_{L1} = 100\Omega$, $\underline{Z}_{L2} = 80\Omega$, $L_1 = 0{,}125 \cdot \lambda$, $L_2 = 0{,}725 \cdot \lambda$, and $f = 20kHz$

1) Determine the input impedance $\underline{Z}_E$.

2) Determine the internal impedance $\underline{Z}_i$ of the voltage source for the case that maximum active power P_0 is converted to heat at $\underline{Z}_0$.

Exercise 25:

An ideal operational amplifier works as an inverting amplifier and is biased with the voltages U_B. The voltage $u_e(t)$ is applied to the input terminals. The voltage $u_a(t)$ can be measured at the output terminals without load.



The bipolar voltage supply has the value U_B=15V. The resistance R_0 = 100kΩ.

1) Size the resistance R_1 for a voltage amplification of |A| = 50.

Now a sinusoidal input voltage u_e(t) is applied. The peak amplitude $\hat{u}_e$ = 0,4V.

2) Draw to scale the time-dependent course of the output voltage $u_a(t)$ in the interval $0 < \omega t < 4\pi$.