Musterlösungen zu den Übungsaufgaben

Grundgebiete der Informationstechnik 1

Version 2.0.3, Datum 16. Mai 2001

Falls bei der Durchsicht Fehler auffallen, bitte ich, mich darauf aufmerksam zu machen, damit sie umgehend beseitigt werden können.

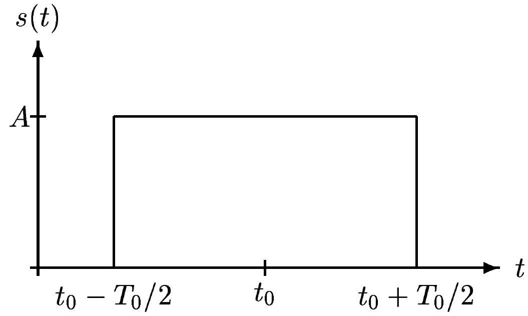
Duisburg, 16. Mai 2001

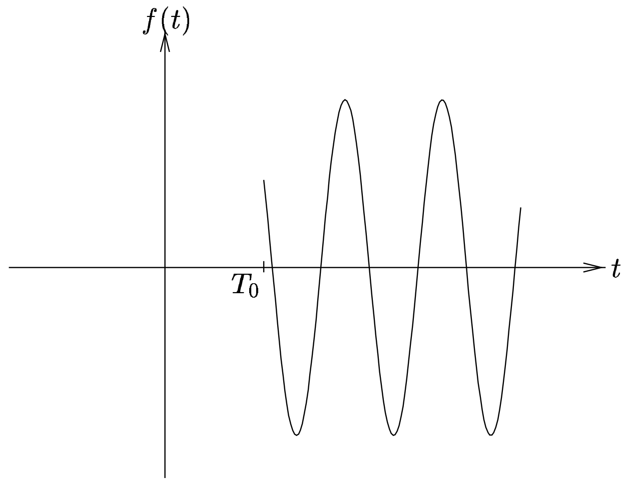
Prof. Dr.-Ing. I. Willms
Dipl.-Ing. F. Gockel

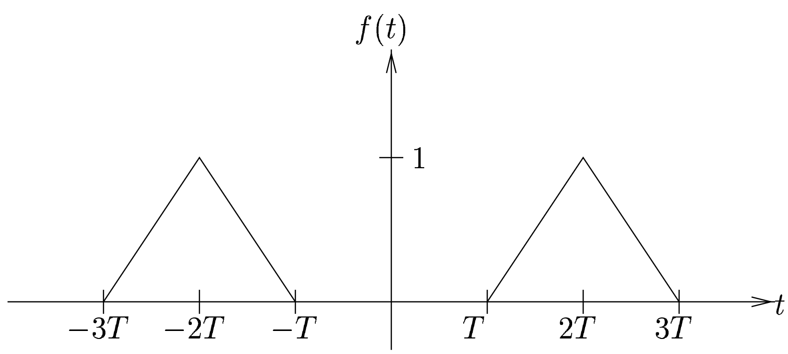
Fachgebiet Nachrichtentechnik

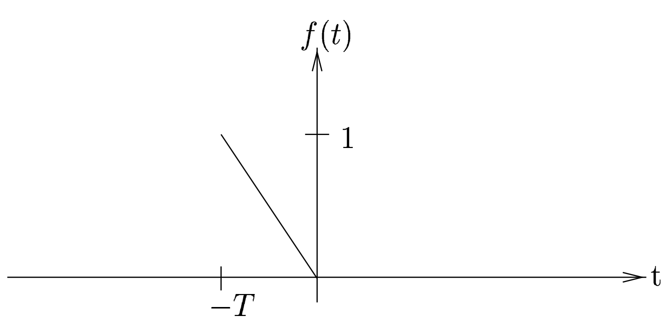
Einführungsaufgabe:

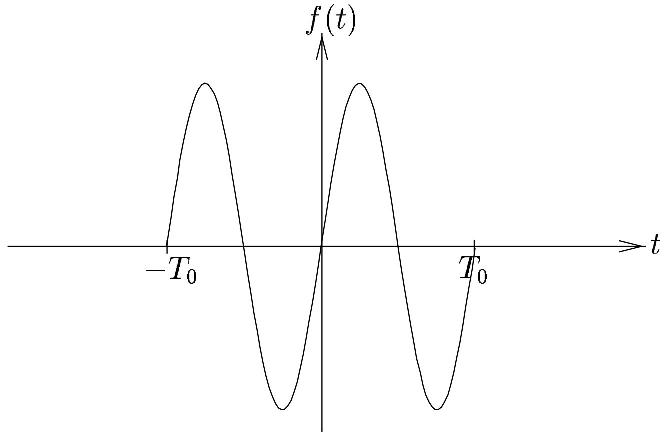
1.

(a)


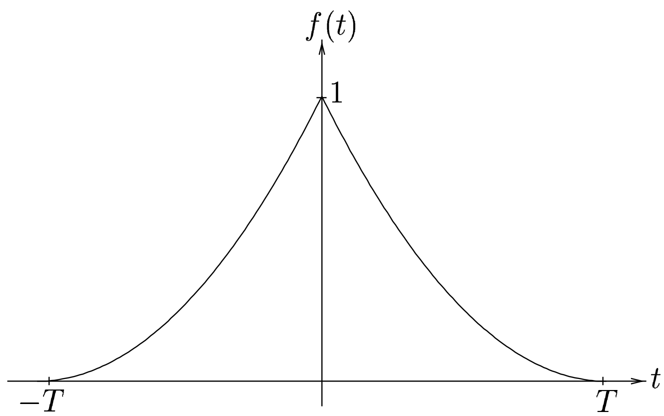
(b)


(c)


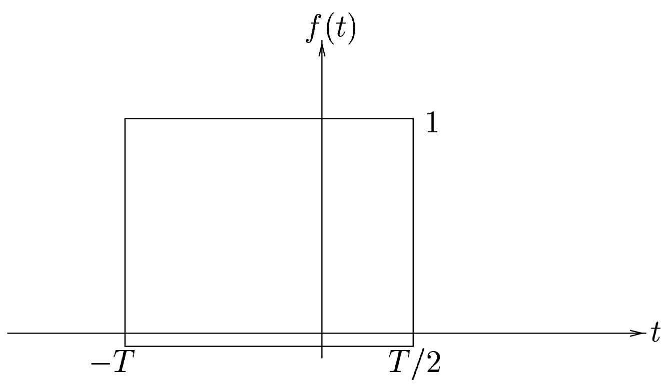
(d)




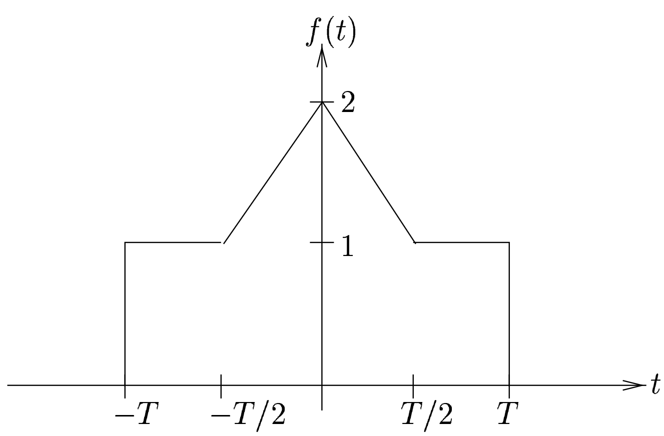
(e)



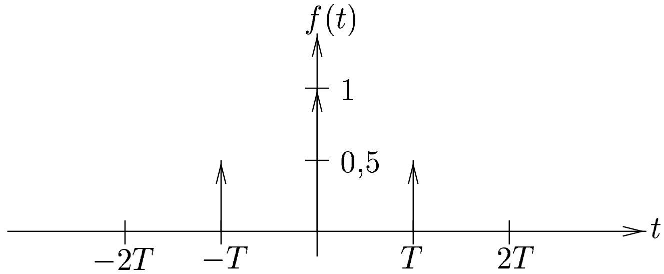
(f)



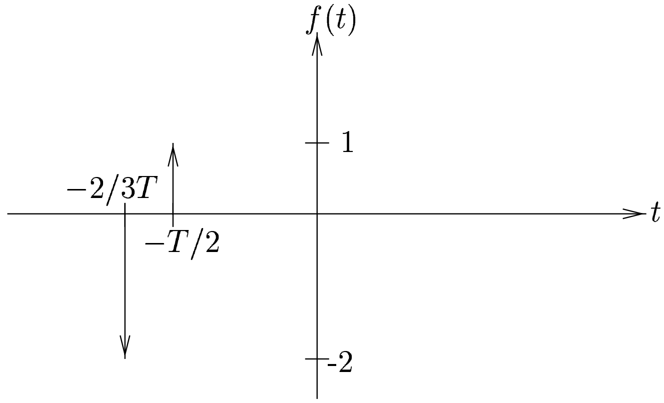
(g)



(h)



(i)



(j)

2.

$$s(t) = \mathrm{r}\left(\frac{t-T}{T}\right)\mathrm{rect}\left(\frac{t-1,5T}{T}\right)+\frac{1}{2}\mathrm{rect}\left(\frac{t-3,5T}{T}\right) + \mathrm{r}\left(\frac{-(t-6T)}{T}\right)\mathrm{rect}\left(\frac{t-5,5T}{T}\right)$$

oder

$$s(t) = 2,5\Lambda\left(\frac{t-3,5T}{2,5T}\right)\left[1-\mathrm{rect}\left(\frac{t-3,5T}{3T}\right)\right]+\frac{1}{2}\mathrm{rect}\left(\frac{t-3,5T}{T}\right)$$

oder

$$s(t) = \mathrm{r}\left(\frac{t-T}{T}\right)\varepsilon(-(t-2T))+\frac{1}{2}\mathrm{rect}\left(\frac{t-3,5T}{T}\right) + \mathrm{r}\left(\frac{-(t-6T)}{T}\right)\varepsilon(t-5T)$$

Aufgabe 1:

a)

$$
\begin{aligned}
c_n &= \frac{1}{T}\int\limits_{-T/2}^{T/2} s(t)\mathrm{e}^{-jn\omega_0 t}dt = \frac{1}{T}\int\limits_{-T_i/2}^{T_i/2} A\mathrm{e}^{-jn\omega_0 t}dt \\
&= \frac{A}{T}\left[-\frac{\mathrm{e}^{-jn\omega_0 t}}{jn\omega_0}\right]_{T_i/2}^{T_i/2} = \frac{A}{T}\frac{-\mathrm{e}^{-jn\omega_0 T_i/2}+\mathrm{e}^{jn\omega_0 T_i/2}}{jn\omega_0} \\
&= \frac{A}{Tn\omega_0}\frac{\mathrm{e}^{jn\omega_0 T_i/2}-\mathrm{e}^{-jn\omega_0 T_i/2}}{j} \qquad \text{mit } \sin(x) = \frac{\mathrm{e}^{jx}-\mathrm{e}^{-jx}}{2j} \\
&= \frac{2A}{Tn\omega_0}\sin\left(n\omega_0\frac{T_i}{2}\right) = \frac{AT_i}{T}\mathrm{si}\left(n\omega_0\frac{T_i}{2}\right)
\end{aligned}
$$

gerades Signal ⇒ reelle Koeffizienten

b)

$$
\begin{aligned}
c_n &= \frac{1}{T}\int\limits_{-T/2}^{T/2} s(t)\mathrm{e}^{-jn\omega_0 t}dt = \frac{1}{T}\int\limits_{-T/4-T_i/2}^{-T/4+T_i/2} -A\mathrm{e}^{-jn\omega_0 t}dt + \frac{1}{T}\int\limits_{T/4-T_i/2}^{T/4+T_i/2} A\mathrm{e}^{-jn\omega_0 t}dt \\
&= \frac{1}{T}\left(-A\left[-\frac{\mathrm{e}^{-jn\omega_0 t}}{jn\omega_0}\right)_{-T/4-T_i/2}^{-T/4+T_i/2} + A\left[-\frac{\mathrm{e}^{-jn\omega_0 t}}{jn\omega_0}\right]_{T/4-T_i/2}^{T/4+T_i/2}\right] \\
&= -\frac{2A}{T}\mathrm{e}^{jn\omega_0 T/4}\frac{\sin(n\omega_0 T_i/2)}{n\omega_0} + \frac{2A}{T}\mathrm{e}^{-jn\omega_0 T/4}\frac{\sin(n\omega_0 T_i/2)}{n\omega_0} \\
&= \frac{2A}{T}\left(-\mathrm{e}^{jn\omega_0 T/4}+\mathrm{e}^{-jn\omega_0 T/4}\right)\frac{\sin(n\omega_0 T_i/2)}{n\omega_0} \\
&= \frac{AT_i}{T}\mathrm{si}(n\omega_0 T_i/2)(-2j)\sin(n\omega_0 T/4), \quad \omega_0 = \frac{2\pi}{T} \\
&= \frac{AT_i}{T}\mathrm{si}\left(n\pi\frac{T_i}{T}\right)(-2j)\sin\left(n\frac{\pi}{2}\right)
\end{aligned}
$$

ungerades Signal ⇒ imaginäre Koeffizienten.

Aufgabe 2:

$$
\begin{aligned}
c_n &= \frac{1}{T}\int_{-T/2}^{T/2} s(t)\mathrm{e}^{-jn\omega_0 t}dt = \frac{A}{T}\int_{-T/2}^{T/2}\cos\left(\frac{\pi t}{T}\right)\mathrm{e}^{-jn\omega_0 t}dt \\
&= \frac{A}{T}\int_{-T/2}^{T/2}\frac{\mathrm{e}^{j\pi t/T}+\mathrm{e}^{-j\pi t/T}}{2}\mathrm{e}^{-j2\pi nt/T}dt \\
&= \frac{A}{2T}\int_{-T/2}^{T/2}\left(\mathrm{e}^{-j(2n-1)\pi t/T}+\mathrm{e}^{-j(2n+1)\pi t/T}\right)dt \\
&= \frac{A}{2T}\left[\frac{\mathrm{e}^{-j(2n-1)\pi t/T}}{-j(2n-1)\pi/T}+\frac{\mathrm{e}^{-j(2n+1)\pi t/T}}{-j(2n+1)\pi/T}\right]_{T/2}^{T/2} \\
&= \frac{A}{2T}\left(\frac{\mathrm{e}^{-j(2n-1)\pi/2}-\mathrm{e}^{j(2n-1)\pi/2}}{-j(2n-1)\pi/T}+\frac{\mathrm{e}^{-j(2n+1)\pi/2}-\mathrm{e}^{j(2n+1)\pi/2}}{-j(2n+1)\pi/T}\right) \\
&= \frac{A}{2T}\left(\frac{-2j\sin[(2n-1)\pi/2]}{-j(2n-1)\pi/T}+\frac{-2j\sin[(2n+1)\pi/2]}{-j(2n+1)\pi/T}\right) \\
&= A\frac{\sin[(2n-1)\pi/2]}{(2n-1)\pi}+A\frac{\sin[(2n+1)\pi/2]}{(2n+1)\pi} \\
&\quad \sin(x\pm y)=\sin x\cos y\pm\cos x\sin y \\
&= A\frac{\sin(n\pi)\cos(\pi/2)-\cos(n\pi)\sin(\pi/2)}{(2n-1)\pi}+ \\
&\qquad A\frac{\sin(n\pi)\cos(\pi/2)+\cos(n\pi)\sin(\pi/2)}{(2n+1)\pi} \\
&= -A\frac{\cos(n\pi)}{(2n-1)\pi}+A\frac{\cos(n\pi)}{(2n+1)\pi}=\frac{A}{\pi}\cos(n\pi)\left(\frac{1}{2n+1}-\frac{1}{2n-1}\right) \\
&= \frac{A}{\pi}\cos(n\pi)\frac{2n-1-(2n+1)}{(2n+1)(2n-1)} = -\frac{2A\cos(n\pi)}{(4n^2-1)\pi} \\
&= -\frac{2A(-1)^n}{(4n^2-1)\pi}=\frac{2A(-1)^{n+1}}{(4n^2-1)\pi} \qquad \text{mit } \cos(n\pi)=(-1)^n
\end{aligned}
$$

Aufgabe 3:

$$
\begin{aligned}
S(\omega) &= \mathcal{F}\{s(t)\} = \int\limits_{-\infty}^{\infty} s(t)\mathrm{e}^{-j\omega t}dt \\
&= \int\limits_{-T/2}^{T/2} 1 \cdot e^{-j\omega t}dt = \left[\frac{\mathrm{e}^{-j\omega t}}{-j\omega}\right]_{-T/2}^{T/2} \\
&= \frac{-\left(\mathrm{e}^{j\frac{1}{2}\omega T} - \mathrm{e}^{-j\frac{1}{2}\omega T}\right)}{-j\omega} = \frac{2j\sin(\frac{1}{2}\omega T)}{j\omega} \\
&= \frac{T\sin(\frac{1}{2}\omega T)}{\frac{1}{2}\omega T} = T\mathrm{si}(\frac{1}{2}\omega T)
\end{aligned}
$$

Aufgabe 4:

$$s(t) = A\left[\mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right) - \mathrm{rect}\left(\frac{t-\frac{3}{2}T}{T}\right)\right]$$

Verschiebungssatz:

$$s(t-t_0) \overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} S(\omega)e^{-j\omega t_0}$$

$$\begin{aligned}\Rightarrow s(t) \overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} S(\omega) &= AT\left[\mathrm{si}\left(\omega\frac{T}{2}\right)e^{-j\frac{1}{2}\omega T} - \mathrm{si}\left(\omega\frac{T}{2}\right)e^{-j\frac{3}{2}\omega T}\right] \\ &= AT\mathrm{si}\left(\frac{1}{2}\omega T\right)\left[e^{-j\frac{1}{2}\omega T} - e^{-j\frac{3}{2}\omega T}\right]\end{aligned}$$

Aufgabe 5:

$$s(t) \overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} S(\omega) = j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0) + \pi\delta(\omega - \omega_0)$$

Aufgabe 6:

1.

$$
\begin{aligned}
s(t) &= \operatorname{rect}\left(\frac{t}{T}\right) * \operatorname{rect}\left(\frac{t}{T}\right) \\
\mathcal{F} &\ \updownarrow \\
S(\omega) &= T\operatorname{si}\left(\omega\frac{T}{2}\right) \cdot T\operatorname{si}\left(\omega\frac{T}{2}\right) \\
&= T^2\operatorname{si}^2\left(\omega\frac{T}{2}\right)
\end{aligned}
$$

2.

$$
\begin{aligned}
s(t) &= \operatorname{si}\left(\pi\frac{t}{T}\right) * \operatorname{si}\left(\pi\frac{t}{T}\right) \\
\mathcal{F} &\ \updownarrow \\
S(\omega) &= T\operatorname{rect}\left(\frac{\omega}{2\pi/T}\right) T\operatorname{rect}\left(\frac{\omega}{2\pi/T}\right) \\
&= T^2\operatorname{rect}\left(\frac{\omega}{2\pi/T}\right)
\end{aligned}
$$

Die si-Funktion reproduziert sich bei der Faltung selbst.

Aufgabe 7:

1.

$$
\begin{aligned}
s(t) &= \varepsilon(t - t_0), \quad t_0 > 0 \\
&\mathcal{L}\ \circ\!\!-\!\!\bullet \\
S_L(p) &= \int_0^\infty s(t)e^{-pt}dt, \qquad p = \sigma + j\omega \\
&= \int_0^\infty \varepsilon(t - t_0)e^{-pt}dt \\
&= \int_{t_0}^\infty e^{-pt}dt = \left[-\frac{1}{p}e^{-pt}\right]_{t_0}^\infty \\
&= \frac{1}{p}e^{-pt_0}, \qquad \sigma > 0
\end{aligned}
$$

2.

$$
\begin{aligned}
s(t) &= \varepsilon(t - t_0) - \varepsilon(t - 2t_0), \qquad t_0 > 0 \\
&\mathcal{L}\ \circ\!\!-\!\!\bullet \\
S_L(p) &= \frac{1}{p}e^{-pt_0} - \frac{1}{p}e^{-2pt_0} = \frac{e^{-pt_0} - e^{-2pt_0}}{p}, \qquad \sigma > 0;
\end{aligned}
$$

3.

$$
\begin{aligned}
s(t) &= \operatorname{rect}\left(\frac{t - \frac{T}{2}}{T}\right), \qquad T > 0 \\
&\mathcal{L}\ \circ\!\!-\!\!\bullet \\
S_L(p) &= \int_0^\infty \operatorname{rect}\left(\frac{t - \frac{T}{2}}{T}\right) e^{-pt}dt \\
&= \int_0^T e^{-pt}dt = \left[\frac{e^{-pt}}{-p}\right]_0^T = \frac{1}{p} - \frac{e^{-pT}}{p}, \qquad \sigma > 0
\end{aligned}
$$

4.

$$
s(t) = e^{\alpha t}\varepsilon(t), \qquad \alpha > 0
$$

$$\mathcal{L} \,\circ\!\!-\!\!\bullet$$

$$\begin{aligned} S_L(p) &= \int_0^\infty e^{\alpha t} e^{-pt} dt = \int_0^\infty e^{(\alpha - p)t} dt \\ &= \left[\frac{e^{(\alpha-p)t}}{\alpha - p} \right]_0^\infty = \frac{1}{p - \alpha}, \qquad \sigma > \alpha \end{aligned}$$

5.

$$s(t) = \varepsilon(t) e^{\alpha t} \cos(\omega_0 t), \qquad \alpha > 0$$

$$\mathcal{L} \,\circ\!\!-\!\!\bullet$$

$$\begin{aligned} S_L(p) &= \int_0^\infty e^{\alpha t} \cos(\omega_0 t) e^{-pt} dt \\ &= \frac{1}{2} \int_0^\infty e^{\alpha t} \left(e^{j\omega_0} + e^{-j\omega_0} \right) e^{-pt} dt \\ &= \frac{1}{2} \left(\frac{1}{p - \alpha - j\omega_0} + \frac{1}{p - \alpha + j\omega_0} \right), \qquad \sigma > \alpha \end{aligned}$$

Aufgabe 8:

1.

$$
\begin{aligned}
s(t) &= t\varepsilon(t) \\
&\mathcal{L} \\
S_L(p) &= \int_0^\infty t e^{-pt} dt \\
&= \left[\frac{e^{-pt}}{-p} \cdot t\right]_0^\infty + \int_0^\infty \frac{e^{-pt}}{p} dt = 0 + \left[\frac{e^{-pt}}{-p^2}\right]_0^\infty = \frac{1}{p^2}
\end{aligned}
$$

2.

$$
\begin{aligned}
s(t) &= (t + t_0)\varepsilon(t) \\
&\mathcal{L} \\
S_L(p) &= \int_0^\infty (t + t_0) e^{-pt} dt \\
&= \int_0^\infty t e^{-pt} dt + t_0 \int_0^\infty e^{-pt} dt = \frac{1}{p^2} + \frac{t_0}{p}
\end{aligned}
$$

Aufgabe 9:

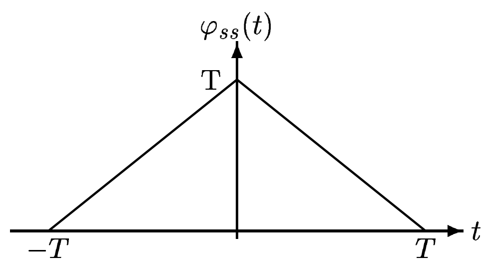
$$s(t) = \begin{cases} a\sin(\omega_0 t) & 0 \le t \le \frac{\pi}{\omega_1} \\ 0 & \text{sonst} \end{cases}$$

$$\mathcal{L} \circ\!\!-\!\!\bullet$$

$$\begin{aligned}
S_L(p) &= \int_0^{\frac{\pi}{\omega_1}} a\sin(\omega_0 t) \\
&= \int_0^{\frac{\pi}{\omega_1}} \frac{a}{2j}\left(e^{-(p-j\omega_0 t)t} - e^{-(p+j\omega_0)t}\right) dt \\
&= \frac{a}{2j}\left[\frac{e^{-(p-j\omega_0)t}}{-(p-j\omega_0)} - \frac{e^{-(p+j\omega_0)t}}{-(p+j\omega_0)}\right]_0^{\frac{\pi}{\omega_1}} \\
&= \frac{a}{2j}\left[\frac{e^{-(p-j\omega_0)\frac{\pi}{\omega_1}}}{-(p-j\omega_0)} - \frac{e^{-(p+j\omega_0)\frac{\pi}{\omega_1}}}{-(p+j\omega_0)} - \frac{1}{-(p-j\omega_0)} + \frac{1}{-(p+j\omega_0)}\right] \\
&= \frac{a}{2j}\left[\frac{-(p+j\omega_0)e^{-(p-j\omega_0)\frac{\pi}{\omega_1}} + (p-j\omega_0)e^{-(p+j\omega_0)\frac{\pi}{\omega_1}} + j2\omega_0}{(p-j\omega_0)(p+j\omega_0)}\right] \\
&= \frac{a}{2j}\left[\frac{-e^{-p\frac{\pi}{\omega_1}}\left(p\mathrm{e}^{j\pi\frac{\omega_0}{\omega_1}} - p\mathrm{e}^{-j\pi\frac{\omega_0}{\omega_1}} + j\omega_0\mathrm{e}^{j\pi\frac{\omega_0}{\omega_1}} + j\omega_0\mathrm{e}^{-j\pi\frac{\omega_0}{\omega_1}}\right) + j2\omega_0}{(p-j\omega_0)(p+j\omega_0)}\right] \\
&= \frac{a}{2j}\left[\frac{-e^{-p\frac{\pi}{\omega_1}}\left(j2p\sin(\pi\frac{\omega_0}{\omega_1}) + j2\omega_0\cos(\pi\frac{\omega_0}{\omega_1})\right) + j2\omega_0}{p^2+\omega_0^2}\right] \\
&= \frac{a\omega_0 - a\mathrm{e}^{-p\frac{\pi}{\omega_1}}\left(p\sin(\pi\frac{\omega_0}{\omega_1}) + \omega_0\cos(\pi\frac{\omega_0}{\omega_1})\right)}{p^2+\omega_0^2}
\end{aligned}$$

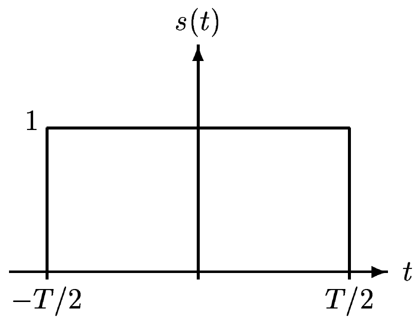
Aufgabe 10:

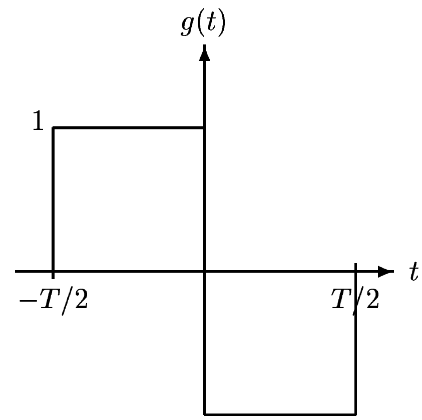
$$
\begin{aligned}
\varphi_{ss}(t) &= s(-t) * s^*(t) \\
&= \int\limits_{-\infty}^{\infty} \operatorname{rect}\left(\frac{-\tau}{T}\right) \operatorname{rect}\left(\frac{t-\tau}{T}\right) d\tau \\
&= \int\limits_{-\infty}^{\infty} \operatorname{rect}\left(\frac{\tau}{T}\right) \operatorname{rect}\left(\frac{t-\tau}{T}\right) d\tau \\
&= \begin{cases} \int\limits_{-T}^{t} d\tau & für \quad -T \le t \le 0 \\ \int\limits_{-T}^{0} d\tau - \int\limits_{0}^{t} d\tau & für \quad 0 \le t \le T \end{cases} \\
&= \begin{cases} t + T & für \quad -T \le t \le 0 \\ T - t & für \quad 0 \le t \le T \end{cases} = T\left(1 - \left|\frac{t}{T}\right|\right) = T\Lambda\left(\frac{t}{T}\right)
\end{aligned}
$$



Aufgabe 11:

1.



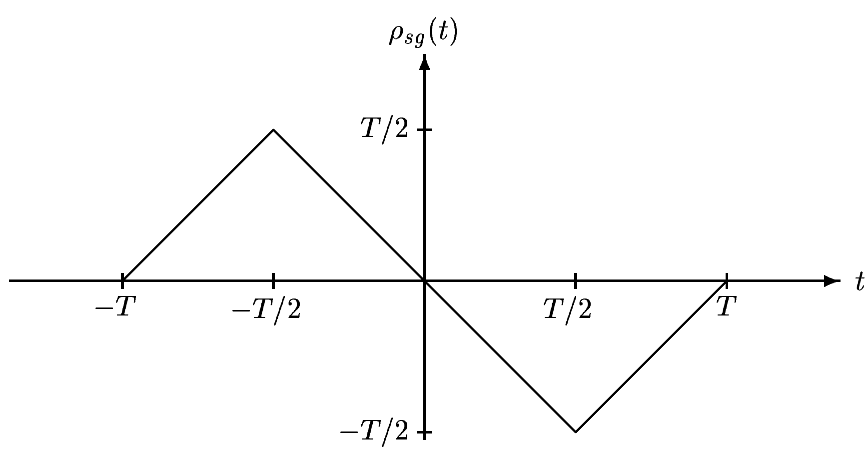


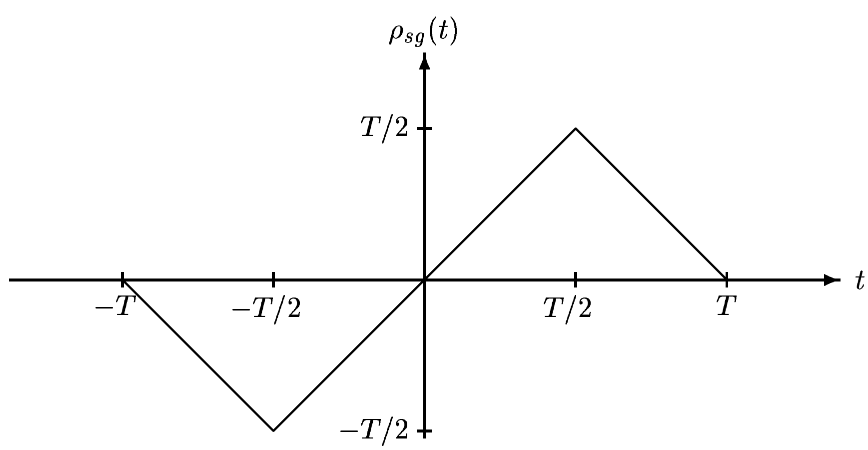
2.

$$\rho_{sg}(t) = s(-t) * g^*(t) = s(t) * g^*(t)$$

$$= \Lambda\left(\frac{t+T/2}{T/2}\right) - \Lambda\left(\frac{t-T/2}{T/2}\right)$$

$$\rho_{gs}(t) = -\rho_{sg}(t) = -\Lambda\left(\frac{t+T/2}{T/2}\right) + \Lambda\left(\frac{t-T/2}{T/2}\right)$$





3.

$$\rho_{sg}(t) = \left[\mathrm{rect}\left(\frac{t}{T/2}\right) * \mathrm{rect}\left(\frac{t}{T/2}\right)\right] * \left[\delta(t-\frac{T}{2}) - \delta(t+\frac{T}{2})\right]$$

$$\mathcal{F} \circ\!\!-\!\!\bullet$$

$$\begin{aligned} S_{sg}(\omega) &= \left(\frac{T}{2}\mathrm{si}(\omega\frac{T}{4})\right)^2 \cdot \left(e^{-j\omega T/2} - e^{j\omega T/2}\right) \\ &= -2j\left(\frac{T}{2}\mathrm{si}(\omega\frac{T}{4})\right)^2 \cdot \sin(\omega\frac{T}{2}) \end{aligned}$$

Bei $\rho_{gs}(t)$ wechseln nur die Vorzeichen bei der δ-Funktion:

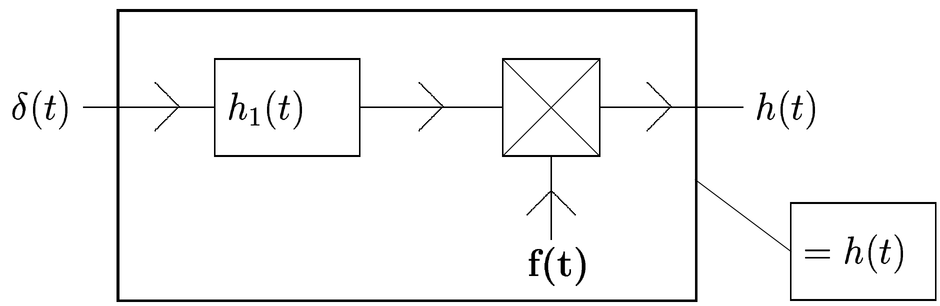
$$S_{gs}(\omega) = 2j\left(\frac{T}{2}\mathrm{si}(\omega\frac{T}{4})\right)^2 \cdot \sin(\omega\frac{T}{2})$$

Aufgabe 12:

a) Linearität: wenn $s(t) = a \cdot \delta(t)$, muß gelten $g(t) = a \cdot h(t)$.

b) Zeitinvarianz: wenn $s(t) = \delta(t - t_0)$, muß gelten $g(t) = h(t - t_0)$.

1. $h(t) = f(t) \cdot h_1(t)$



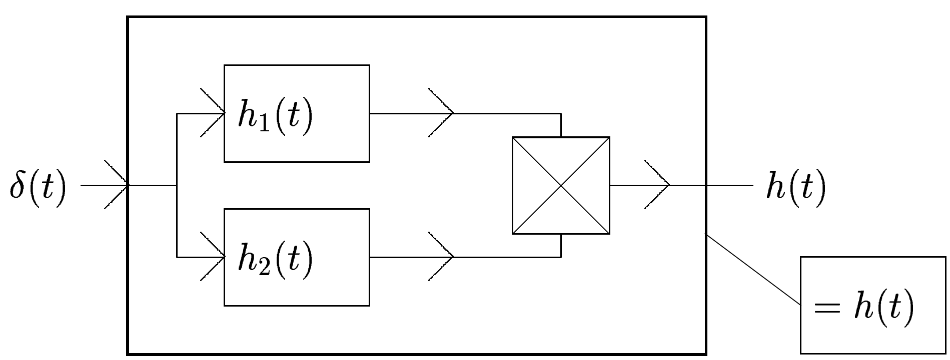
a) $s(t) = a \cdot \delta(t) \Rightarrow g(t) = a \cdot h_1(t) \cdot f(t) = a \cdot h(t)$

$\Rightarrow$ **LINEAR**

b) $s(t) = \delta(t - t_0) \Rightarrow g(t) = h_1(t - t_0) \cdot f(t) \neq h(t - t_0)$

$\Rightarrow$ **ZEITVARIANT**

2. $h(t) = h_1(t) \cdot h_2(t)$



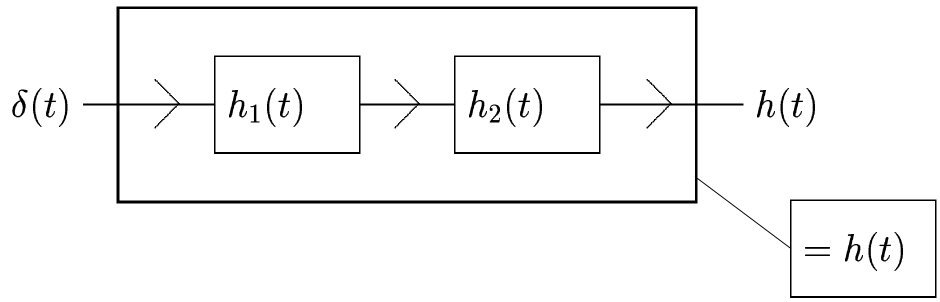
a) $s(t) = a \cdot \delta(t) \Rightarrow g(t) = a \cdot h_1(t) \cdot h_2(t) = a^2 \cdot h(t) \neq a \cdot H(t)$

$\Rightarrow$ **NICHTLINEAR**

b) $s(t) = \delta(t - t_0) \Rightarrow g(t) = h_1(t - t_0) \cdot h_2(t - t_0) = h(t - t_0)$

$\Rightarrow$ **ZEITINVARIANT**

3. $h(t) = h_1(t) * h_2(t)$



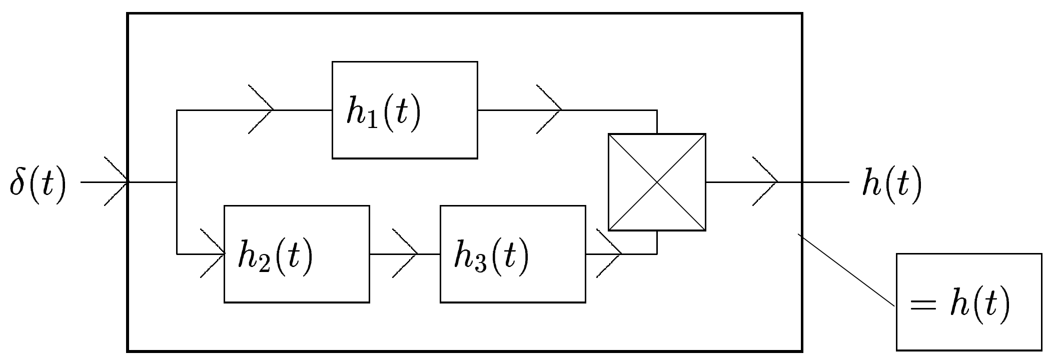
a) $s(t) = a \cdot \delta(t) \Rightarrow g(t) = a \cdot h_1(t) * h_2(t) = a \cdot h(t)$

$\Rightarrow$ **LINEAR**

b) $s(t) = \delta(t - t_0) \Rightarrow g(t) = h_1(t - t_0) * h_2(t - t_0) = \delta(t - t_0) * h(t) = h(t - t_0)$

$\Rightarrow$ **ZEITINVARIANT**

4. $h(t) = h_1 \cdot h_2(t) * h_3(t)$



a) $s(t) = a \cdot \delta(t) \Rightarrow g(t) = a \cdot h_1(t) \cdot a \cdot h_2(t) * h_3(t) = a^2 \cdot h(t) \neq a \cdot h(t)$

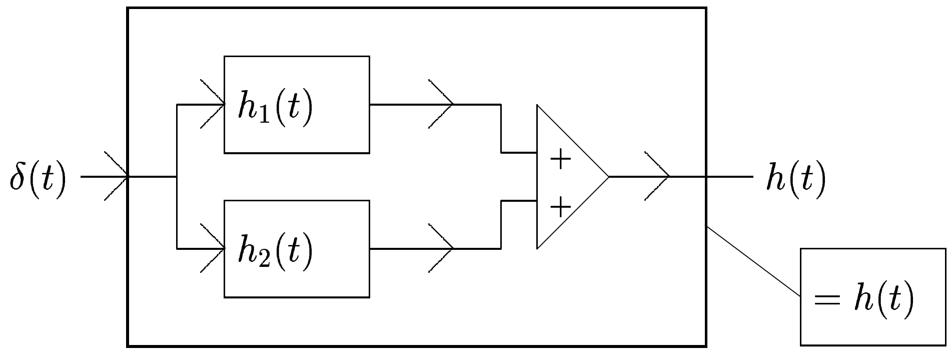
$\Rightarrow$ **NICHTLINEAR**

b) $s(t) = \delta(t - t_0) \Rightarrow g(t) = h_1(t - t_0) \cdot h_4(t - t_0) = h(t - t_0)$

(Hierbei ist $h_4(t) = h_2(t) * h_3(t)$)

$\Rightarrow$ **ZEITINVARIANT**

5. $h(t) = h_1(t) + h_2(t)$



a) $s(t) = a \cdot \delta(t) \Rightarrow g(t) = a \cdot h_1(t) + a \cdot h_2(t) = a \cdot h(t)$

$\Rightarrow$ **LINEAR**

b) $s(t) = \delta(t - t_0) \Rightarrow g(t) = h_1(t - t_0) + h_2(t - t_0) = h(t - t_0)$

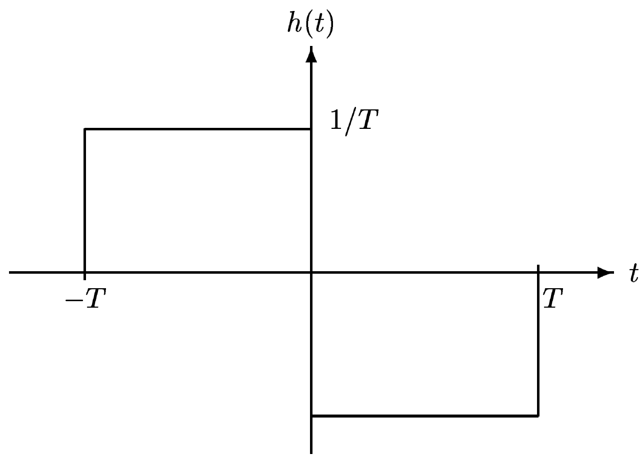
$\Rightarrow$ **ZEITINVARIANT**

Aufgabe 13:

$$h(t) = \frac{d}{dt}e(t)$$

$$h(t) = \frac{d}{dt}\begin{cases} \frac{1}{T}(T+t) & -T \leq t \leq 0 \\ \frac{1}{T}(T-t) & 0 \leq t \leq T \\ 0 & sonst \end{cases} = \begin{cases} \frac{1}{T} & -T \leq t \leq 0 \\ \frac{1}{T} & 0 \leq t \leq T \\ 0 & sonst \end{cases}$$

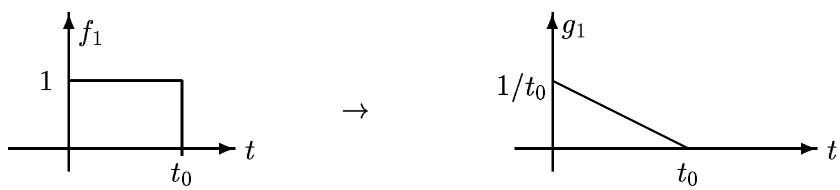
$$h(t) = \text{rect}\left(\frac{t+T/2}{T}\right) - \text{rect}\left(\frac{t-T/2}{T}\right)$$

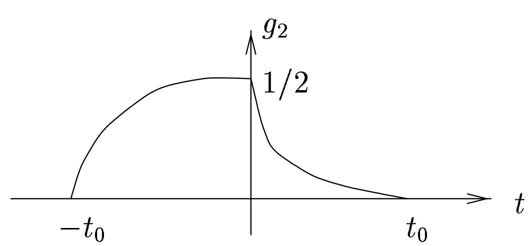


Aufgabe 14:

1. Gesucht: $g_2(t) = f_2(t) * h(t)$

$$\begin{aligned} f_2(t) &= \frac{1}{t_0} \cdot [f_1(t) * f_1(t+t_0)] \quad , g_1(t) = f_1(t) * h(t) \\ \Rightarrow g_2(t) &= \frac{1}{t_0} \cdot [f_1(t) * f_1(t) * f_1(t+t_0)] h(t) \\ &= \frac{1}{t_0} \cdot [f_1(t) * h(t)] * f_1(t+t_0) = \frac{1}{t_0} \cdot g_1(t) * f_1(t+t_0) \end{aligned}$$

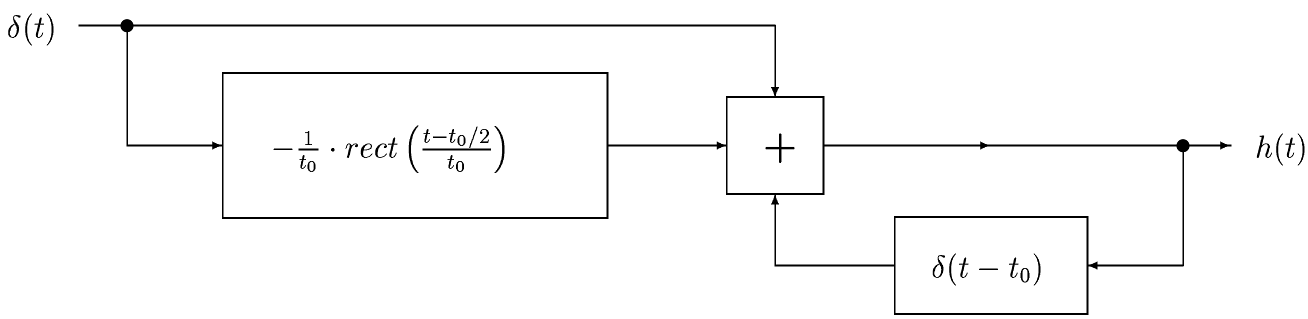




2.

$$\frac{df_1(t)}{dt} * h(t) = \frac{dg_1(t)}{dt} \Rightarrow \frac{df_1}{dt} = \frac{d}{dt}\left[\mathrm{rect}\left(\frac{t - t_0/2}{t_0}\right)\right] = \delta(t) - \delta(t - t_0)$$

$$\begin{aligned} \frac{dg_1(t)}{dt} &= \frac{d}{dt}\left[\left(1 - \frac{t}{t_0}\right) \cdot \mathrm{rect}\left(\frac{t - t_0/2}{t_0}\right)\right] = \frac{d}{dt}\left[\left(f_1(t) \cdot (1 - \frac{t}{t_0}\right)\right] \\ &= \left(1 - \frac{t}{t_0}\right) \cdot (\delta(t) - \delta(t-t_0)) - \frac{1}{t_0} \cdot f_1(t) = -\frac{1}{t_0} \cdot \mathrm{rect}\left(\frac{t - t_0/2}{t_0}\right) + \delta(t) \\ h(t) &= h(t - t_0) - \frac{1}{t_0} \cdot \mathrm{rect}\left(\frac{t - t_0/2}{t_0}\right) + \delta(t) \end{aligned}$$


δ(t)
−1/t0 · rect((t−t0/2)/t0)
+
h(t)
δ(t − t0)

Aufgabe 15:

1.

$$H(\omega) = H_0 \cdot \text{rect}\left(\frac{\omega - \Omega}{2\omega_g}\right) + H_0 \cdot \left(\frac{\omega + \Omega}{2\omega_g}\right)$$

2.

$$H(\omega) = H_0 \cdot \text{rect}\left(\frac{\omega}{2\omega_g}\right) * (\delta(\omega - \Omega) + \delta(\omega + \Omega))$$

3.

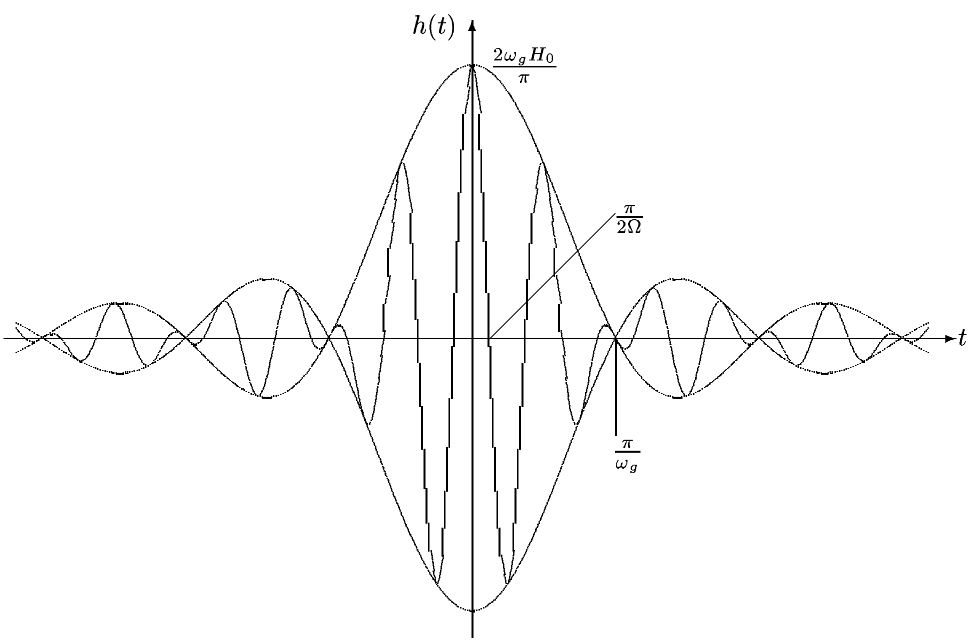
$$H_0 \cdot \text{rect}\left(\frac{\omega}{2\omega_g}\right) \overset{\mathcal{F}}{\bullet\!\!-\!\!\circ} H_0 \frac{\omega_g}{\pi} \text{si}(\omega_g t)$$

$$\delta(\omega - \Omega) + \delta(\omega + \Omega) \overset{\mathcal{F}}{\bullet\!\!-\!\!\circ} \frac{1}{\pi} \cos(\Omega t)$$

$$A(\omega) * B(\omega) \overset{\mathcal{F}}{\bullet\!\!-\!\!\circ} 2\pi \cdot a(t) \cdot b(t)$$

$$\Rightarrow h(t) = H_0 \frac{2\omega_g}{\pi} \cdot \text{si}(\omega_g t) \cdot \cos(\Omega t)$$

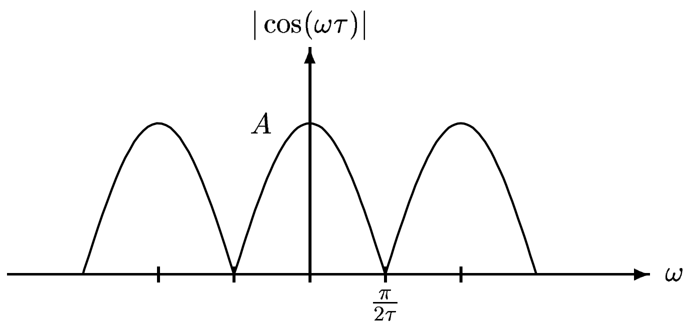
Die si-Funktion stellt die Einhüllende dar!



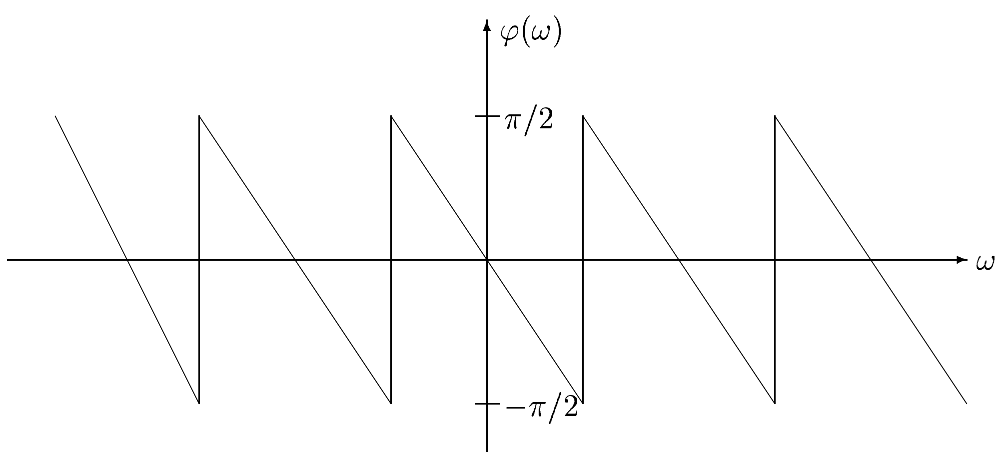
Aufgabe 16:

$H(\omega) = \cos(\omega\tau)e^{-j\omega t}$
Aus der Polardarstellung sieht man:

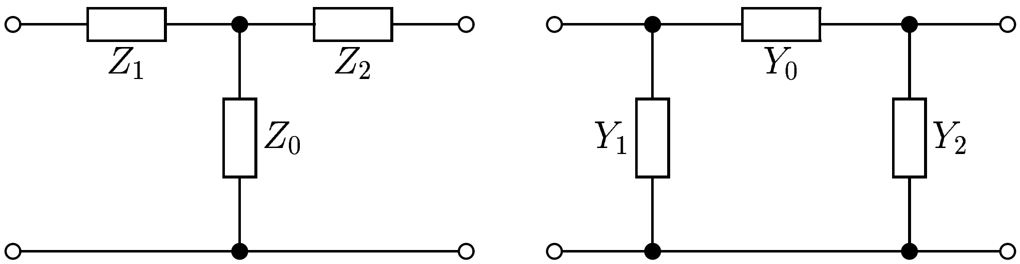
$$
\begin{aligned}
|H(\omega)| &= |H_1(\omega)| \cdot |H_2(\omega)| \\
&= |\cos(\omega\tau)| \cdot 1 = |\cos(\omega\tau)|
\end{aligned}
$$



$$
\begin{aligned}
\text{Phase} \quad \varphi(\omega) &= \varphi_1(\omega) + \varphi_2(\omega) \\
\varphi_1(\omega) &= \begin{cases} 0 & \text{für } \cos(\omega\tau) > 0 \\ \pi & \text{für } \cos(\omega\tau) < 0 \end{cases} \\
\varphi_2(\omega) &= -\omega\tau \\
\Rightarrow \varphi(\omega) &= \begin{cases} -\omega\tau & \text{für } \cos(\omega\tau) > 0 \\ \pi - \omega\tau & \text{für } \cos(\omega\tau) < 0 \end{cases}
\end{aligned}
$$



Aufgabe 23:



T-Schaltung   Π-Schaltung

1) T-Ersatzschaltung

$$U_1 = Z_{11}I_1 + Z_{12}I_2$$

$$U_2 = Z_{21}I_1 + Z_{22}I_2$$

$$I_2 = 0 \Rightarrow U_1 = Z_1I_1 + Z_0I_1 = (Z_1 + Z_0)I_1 = Z_{11}I_1$$

$$Z_{11} = Z_1 + Z_0$$

$$I_1 = 0 \Rightarrow U_2 = Z_2I_2 + Z_0I_2 = (Z_2 + Z_0)I_2 = Z_{22}I_2$$

$$Z_{22} = Z_2 + Z_0$$

$$U_2 = 0 \Rightarrow I_2 = -\frac{Z_{12}}{Z_{22}}I_1 \Rightarrow U_1 = \left(Z_{11} - \frac{Z_{12}^2}{Z_{22}}\right) I_1$$

$$U_1 = \left(Z_1 + \frac{Z_0Z_2}{Z_0 + Z_2}\right) I_1$$

$$Z_{11} - \frac{Z_{12}^2}{Z_{22}} = Z_1 + Z_0 - \frac{Z_{12}^2}{Z_2 + Z_0} = Z_1 + \frac{Z_0Z_2}{Z_0 + Z_2}$$

$$\frac{Z_{12}^2}{Z_2 + Z_0} = Z_0 - \frac{Z_0Z_2}{Z_0 + Z_2} = \frac{Z_0^2 + Z_0Z_2 - Z_0Z_2}{Z_0 + Z_2}$$

$$Z_{12} = Z_0$$

Somit gilt:

$$Z_0 = Z_{12} \quad Z_1 = Z_{11} - Z_{12} \quad Z_2 = Z_{22} - Z_{12}$$

2) Π-Ersatzschaltung

$$I_1 = Y_{11}U_1 + Y_{12}U_2$$

$$I_2 = Y_{21}U_1 + Y_{22}U_2$$

$$U_1 = 0 \;\Rightarrow\; Y_{11} = Y_1 + Y_0$$

$$U_2 = 0 \;\Rightarrow\; Y_{22} = Y_2 + Y_0$$

$$I_1 = 0 \;\Rightarrow\; Y_{12} = -Y_0$$

Also:

$$Y_0 = -Y_{12} \quad Y_1 = Y_{11} + Y_{12} \quad Y_2 = Y_{22} + Y_{12}$$

Aufgabe 24:

1) T-Schaltung

$$
\begin{aligned}
Z_{W_1} &= \sqrt{Z_{1\mathrm{K}} Z_{1\mathrm{L}}} \\
Z_{1\mathrm{K}} &= Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2} \\
Z_{1\mathrm{L}} &= Z_1 + Z_0 \\
Z_{W_1} &= \sqrt{\left(Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}\right)(Z_1 + Z_0)} \\
&= \sqrt{Z_1^2 + Z_1 Z_0 + (Z_1 + Z_0)\frac{Z_0 Z_2}{Z_0 + Z_2}} \\
Z_{W_2} &= \sqrt{Z_{2\mathrm{K}} Z_{2\mathrm{L}}} \\
Z_{2\mathrm{K}} &= Z_2 + \frac{Z_0 Z_1}{Z_0 + Z_1} \\
Z_{2\mathrm{L}} &= Z_0 + Z_2 \\
Z_{W_2} &= \sqrt{\left(Z_2 + \frac{Z_0 Z_1}{Z_0 + Z_1}\right)(Z_0 + Z_2)} \\
&= \sqrt{Z_2^2 + Z_0 Z_2 + (Z_0 + Z_2)\frac{Z_0 Z_1}{Z_0 + Z_1}}
\end{aligned}
$$

bei Symmetrie:

$$
\begin{aligned}
Z_{W_1} &= \sqrt{Z_1^2 + 2 Z_0 Z_1} \\
Z_{W_2} &= \sqrt{Z_1^2 + 2 Z_0 Z_1} = Z_{W_1}
\end{aligned}
$$

Zahlenwerte: $Z_{W_1} = 296,65\Omega$, $Z_{W_2} = 370,81\Omega$

2) Π-Schaltung

$$
\begin{aligned}
Z_{1\mathrm{K}} &= \frac{Z_0 Z_1}{Z_0 + Z_1} \\
Z_{1\mathrm{L}} &= \frac{Z_1 (Z_0 + Z_2)}{Z_0 + Z_1 + Z_2} \\
Z_{W_1} &= \sqrt{\frac{Z_0 Z_1^2 (Z_0 + Z_2)}{(Z_0 + Z_1)(Z_0 + Z_1 + Z_2)}}
\end{aligned}
$$

$$Z_{2_K} = \frac{Z_0 Z_2}{Z_0 + Z_2}$$

$$Z_{2L} = \frac{Z_2(Z_0 + Z_1)}{Z_0 + Z_1 + Z_2}$$

$$Z_{W_2} = \sqrt{\frac{Z_0 Z_2^2 (Z_0 + Z_1)}{(Z_0 + Z_2)(Z_0 + Z_1 + Z_2)}}$$

bei Symmetrie:

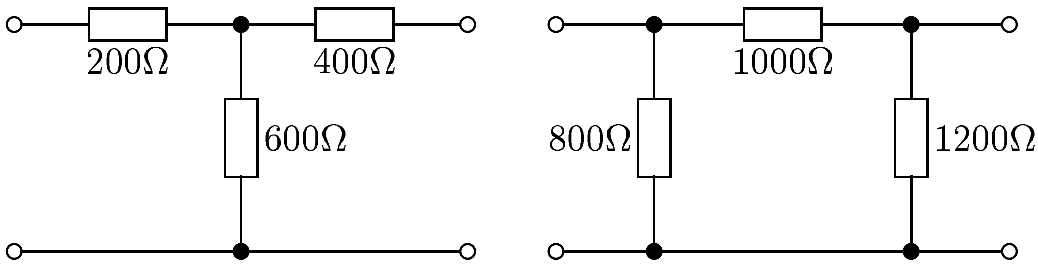
$$Z_{W_1} = \sqrt{\frac{Z_0 Z_1^2}{Z_0 + 2Z_1}}$$

$$Z_{W_2} = \sqrt{\frac{Z_0 Z_1^2}{Z_0 + 2Z_1}} = Z_{W_1}$$

Zahlenwerte: $Z_{W_1} = 79,06\Omega$, $Z_{W_2} = 126,49\Omega$

Aufgabe 25:

Es handelt sich um eine Parallelschaltung aus einer T- und einer Π-Schaltung.



1. Admittanzen
   T-Schaltung:

$$
\begin{aligned}
\det Z &= Z_{11}Z_{22} - Z_{12}Z_{21} \\
&= (Z_1 + Z_0)(Z_2 + Z_0) - Z_0^2 \\
&= Z_1 Z_2 + Z_0 Z_1 + Z_0 Z_2 + Z_0^2 - Z_0^2 \\
&= Z_0 Z_1 + Z_0 Z_2 + Z_1 Z_2 = 440 \cdot 10^3 \Omega^2 \\
Y_{11}^T &= \frac{Z_0 + Z_2}{\det Z} = 2,2727\text{mS} \\
Y_{12}^T &= -\frac{Z_0}{\det Z} = -1,3636\text{mS} \\
Y_{22}^T &= \frac{Z_0 + Z_1}{\det Z} = 1,8181\text{mS}
\end{aligned}
$$

   Π-Schaltung:

$$
\begin{aligned}
Y_{11}^{\Pi} &= Y_0 + Y_1 = 2,25\text{mS} \\
Y_{12}^{\Pi} &= -Y_0 = -1\text{mS} \\
Y_{22}^{\Pi} &= Y_0 + Y_2 = 1.8333\text{mS}
\end{aligned}
$$

   Parallelschaltung:

$$
\begin{aligned}
Y_{11} &= Y_{11}^T + Y_{11}^{\Pi} = 4.5227\text{mS} \\
Y_{12} &= Y_{12}^T + Y_{12}^{\Pi} = -2.3636\text{mS}
\end{aligned}
$$

$$\begin{aligned} Y_{22} &= Y_{22}^T + Y_{22}^\Pi = 3.6514\text{mS} \\ \det Y &= Y_{11}Y_{22} - Y_{12}^2 = 10,928 \cdot 10^{-6}\text{S}^2 \end{aligned}$$

2. Kettenparameter Bestimmung der Kettenparameter aus Impedanzen oder Admittanzen

$$\begin{aligned} I_1 &= Y_{11}U_1 + Y_{12}U_2 \\ I_2 &= Y_{21}U_1 + Y_{22}U_2 \Rightarrow U_1 = -\frac{Y_{22}}{Y_{21}}U_2 + \frac{1}{Y_{21}}I_2 \\ I_1 &= -Y_{11}\frac{Y_{22}}{Y_{21}}U_2 + \frac{Y_{11}}{Y_{21}}I_2 + Y_{12}U_2 = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}U_2 + \frac{Y_{11}}{Y_{21}}I_2 \\ &= -\frac{\det Y}{Y_{21}}U_2 - \frac{Y_{11}}{Y_{21}}(-I_2) \\ U_1 &= \frac{1}{Y_{11}}I_1 - \frac{Y_{12}}{Y_{11}}U_2 \\ &= -\frac{\det Y}{Y_{11}Y_{21}}U_2 + \frac{1}{Y_{21}}I_2 - \frac{Y_{12}}{Y_{11}}U_2 \\ &= -\frac{(Y_{11}Y_{22} + Y_{12}Y_{21}) - Y_{12}Y_{21}}{Y_{11}Y_{21}}U_2 + \frac{1}{Y_{21}}I_2 \\ &= -\frac{Y_{22}}{Y_{21}}U_2 - \frac{1}{Y_{21}}(-I_2) \end{aligned}$$

Durch Vergleich erhält man die Kettenparameter:

$$A_{11} = -\frac{Y_{22}}{Y_{21}} \quad A_{12} = -\frac{1}{Y_{21}} \quad A_{21} = -\frac{\det Y}{Y_{21}} \quad A_{22} = -\frac{Y_{11}}{Y_{21}}$$

In entsprechender Weise ergibt sich:

$$A_{11} = \frac{Z_{11}}{Z_{21}} \quad A_{12} = \frac{\det Z}{Z_{21}} \quad A_{21} = \frac{1}{Z_{21}} \quad A_{22} = \frac{Z_{22}}{Z_{21}}$$

$$\begin{aligned} A_{11} &= -\frac{Y_{22}}{Y_{21}} = 1,5448 \\ A_{12} &= -\frac{1}{Y_{21}} = 423,083\Omega \\ A_{21} &= -\frac{\det Y}{Y_{21}} = 4,6233\text{mS} \end{aligned}$$

$$
\begin{aligned}
A_{22} &= -\frac{Y_{11}}{Y_{21}} = 1.9135 \\
\det A &= A_{11}A_{22} - A_{21}A_{12} = 0.9999 \approx 1
\end{aligned}
$$

3. Kurz- und Leerlaufwiderstände

$$
\begin{aligned}
Z_{1\mathrm{K}} &= \left.\frac{U_1}{I_1}\right|_{U_2=0} = \frac{1}{Y_{11}} = \frac{A_{12}}{A_{22}} = 221,11\Omega \\
Z_{1\mathrm{L}} &= \left.\frac{U_1}{I_1}\right|_{I_2=0} = Z_{11} = \frac{A_{11}}{A_{21}} = 334,13\Omega \\
Z_1 &= \frac{U_1}{I_1} = \frac{A_{11}U_2 - A_{12}I_2}{A_{21}U_2 - A_{22}I_2} \quad , \quad U_2 = -Z_a I_2 \\
&= \frac{A_{11}Z_a + A_{12}}{A_{21}Z_a + A_{22}} = 282,95\Omega
\end{aligned}
$$

4. Wellenwiderstände

$$
\begin{aligned}
Z_{W_1} &= \sqrt{Z_{1\mathrm{K}}Z_{1\mathrm{L}}} = \sqrt{\frac{A_{11}A_{12}}{A_{21}A_{22}}} = 271,81\Omega \\
Z_{W_2} &= \sqrt{Z_{2\mathrm{K}}Z_{2\mathrm{L}}} = \sqrt{\frac{A_{22}A_{12}}{A_{21}A_{11}}} = 336,68\Omega
\end{aligned}
$$

Aufgabe 26:

a)

$$g_W = \operatorname{artanh}\sqrt{\frac{Z_{1K}}{Z_{1L}}}$$

$$Z_{1K} = Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0} = 220\Omega$$

$$Z_{1L} = Z_1 + Z_0 = 400\Omega$$

$$g_W = \operatorname{artanh}\sqrt{0,55} = 0,9541\text{Np}$$

b)

$$g_W = \operatorname{artanh}\sqrt{\frac{Z_{1K}}{Z_{1L}}}$$

$$Z_{1K} = \frac{Z_1 Z_0}{Z_1 + Z_0} = 75\Omega$$

$$Z_{1L} = \frac{Z_1(Z_0 + Z_2)}{Z_0 + Z_1 + Z_2} = 83,3333\Omega$$

$$g_W = \operatorname{artanh}\sqrt{0,9} = 1,8185\text{Np}$$

Aufgabe 27:

Umrechnung der Wellengrößen Z_W, g_W in die Kettenparameter A_{ik}:

$$A_{11} = \cosh g_W \sqrt{\frac{Z_{W_1}}{Z_{W_2}}} = \cosh a_W = 1,3374$$

$$A_{12} = \sinh g_W \sqrt{Z_{W_1} Z_{W_2}} = \sinh a_W Z_W = 373\Omega$$

$$A_{21} = \sinh g_W \sqrt{\frac{1}{Z_{W_1} Z_{W_2}}} = \frac{\sinh a_W}{Z_W} = 2,1145\text{mS}$$

$$A_{22} = \cosh g_W \sqrt{\frac{Z_{W_2}}{Z_{W_1}}} = \cosh a_W = 1,3374$$

$$\cosh a_W = \frac{\mathrm{e}^{a_W} + \mathrm{e}^{-a_W}}{2} = \frac{\mathrm{e}^{0.8} + \mathrm{e}^{-0.8}}{2} = 1,3374$$

$$\sinh a_W = \frac{\mathrm{e}^{a_W} - \mathrm{e}^{-a_W}}{2} = \frac{\mathrm{e}^{0.8} - \mathrm{e}^{-0.8}}{2} = 0,8881$$

$$U_1 = 1,3374 U_2 + 373\Omega(-I_2)$$

$$I_1 = 2,1145\text{mS} U_2 + 1,3374(-I_2)$$

Aufgabe 28:

.1

$$
\begin{aligned}
Z_{1\mathrm{L}} &= Z_C + Z_R = 1000\Omega - j795,8\Omega = 1278\Omega\mathrm{e}^{-j38,51^\circ}\\
Z_{1\mathrm{K}} &= Z_C = -j795,8\Omega = 795,8\Omega\mathrm{e}^{-j90^\circ}\\
Z_{2\mathrm{L}} &= Z_R = 1000\Omega\\
Z_{2K} &= \frac{Z_R Z_C}{Z_R + Z_C} = \frac{795,8\cdot 10^3\Omega^2\mathrm{e}^{-j90^\circ}}{1278\Omega\mathrm{e}^{-j38,51^\circ}} = 622,7\Omega\mathrm{e}^{-j51,49^\circ}\\
Z_{W_1} &= \sqrt{Z_{1\mathrm{K}}Z_{1\mathrm{L}}} = \sqrt{795,8\Omega\mathrm{e}^{-j90^\circ}\cdot 1278\Omega\mathrm{e}^{-j38,51^\circ}} = 1008,5\Omega\mathrm{e}^{-j64,25^\circ}\\
Z_{W_2} &= \sqrt{Z_{2\mathrm{K}}Z_{2\mathrm{L}}} = \sqrt{1000\Omega\cdot 622,7\Omega\mathrm{e}^{-j51,49^\circ}} = 789,1\Omega\mathrm{e}^{-j25,74^\circ}\\
\tanh g_W &= \sqrt{\frac{Z_{1\mathrm{K}}}{Z_{1\mathrm{L}}}} = \sqrt{\frac{795,8\Omega\mathrm{e}^{-j90^\circ}}{1278\Omega\mathrm{e}^{-j38,51^\circ}}} = 0,7891\mathrm{e}^{-j25,74^\circ} = 0,7108 - j0,3427 = z
\end{aligned}
$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}} = z$$

$$\mathrm{e}^x - \mathrm{e}^{-x} = z\mathrm{e}^x + z\mathrm{e}^{-x}$$

$$\mathrm{e}^x(1-z) = \mathrm{e}^{-x}(1+z)$$

$$\mathrm{e}^{2x} = \frac{1+z}{1-z}$$

$$x = \frac{1}{2}\ln\left(\frac{1+z}{1-z}\right)$$

$$g_W = x = \frac{1}{2}\ln\left(\frac{1+z}{1-z}\right) = a_W + jb_W$$

$$1 + z = 1,7108 - j0,3427 = 1,7448\mathrm{e}^{-j11,33^\circ}$$

$$1 - z = 0,2892 + j0,3427 = 0,4484\mathrm{e}^{j49,84^\circ}$$

$$\frac{1+z}{1-z} = 3,8912\mathrm{e}^{-j61,17^\circ}$$

komplexer Logarithmus:

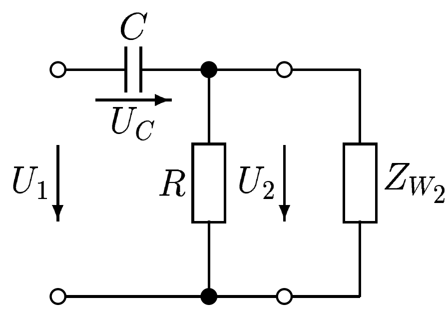
$$\ln(z) = \ln[r(\cos\varphi + j\sin\varphi)] = \ln(r\mathrm{e}^{j\varphi}) = \ln(r) + j(\varphi + 2k\pi)$$

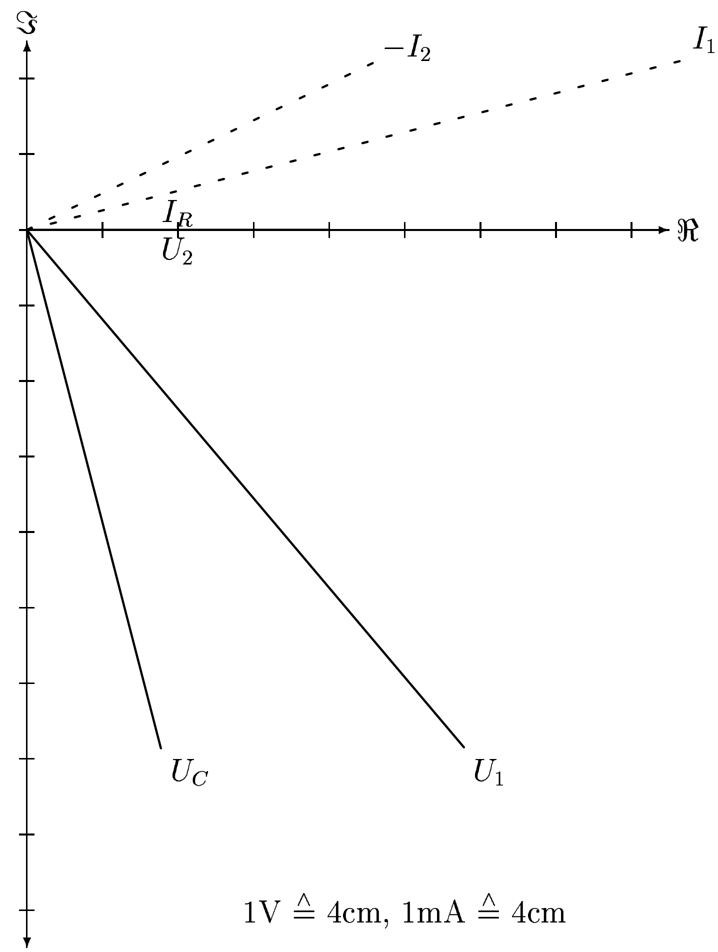
Hauptwert für $k = 0$: $\ln(r\mathrm{e}^{j\varphi}) = \ln(r) + j\varphi$

$$a_W = \frac{1}{2}\ln 3,8912 = 0,6794\text{Np}$$

$$b_W = \frac{1}{2}(-61,17°) = -30,58° \cdot \frac{\pi}{180°} = 0,5337$$

.2





$$-I_2 = \frac{U_2}{Z_{W_2}} = 1,27\text{mAe}^{j25,74°}$$

$$I_R = \frac{U_2}{R} = \frac{1\text{V}}{1000\Omega} = 1\text{mAe}^{j0°}$$

$$I_1 = I_R + (-I_2) = 2,23\text{mAe}^{j14,5°}$$

$$U_C = Z_C I_1 = 1,77\text{Ve}^{-j75,5°}$$

$$U_1 = U_C + U_2 = 2,24\text{Ve}^{-j49,8°}$$

$$Z_{\mathrm{W1}} = \frac{U_1}{I_1} = 1005\Omega \mathrm{e}^{-j64,3^\circ}$$

$$g_B = \ln\left(\frac{U_0/2}{U_2}\sqrt{\frac{Z_{W_2}}{Z_{W_1}}}\right) = \ln\left(\frac{U_1}{U_2}\sqrt{\frac{Z_{W_2}}{Z_{W_1}}}\right)$$

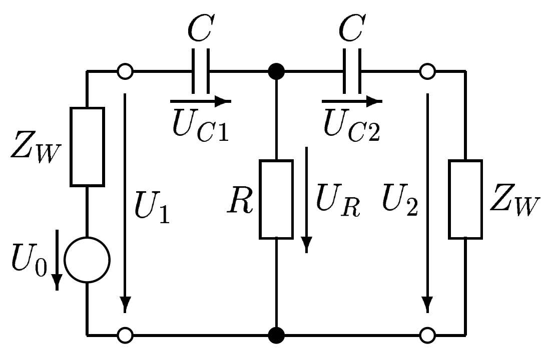
$$g_B = 0,6855 - j0,5327$$

Aufgabe 29:

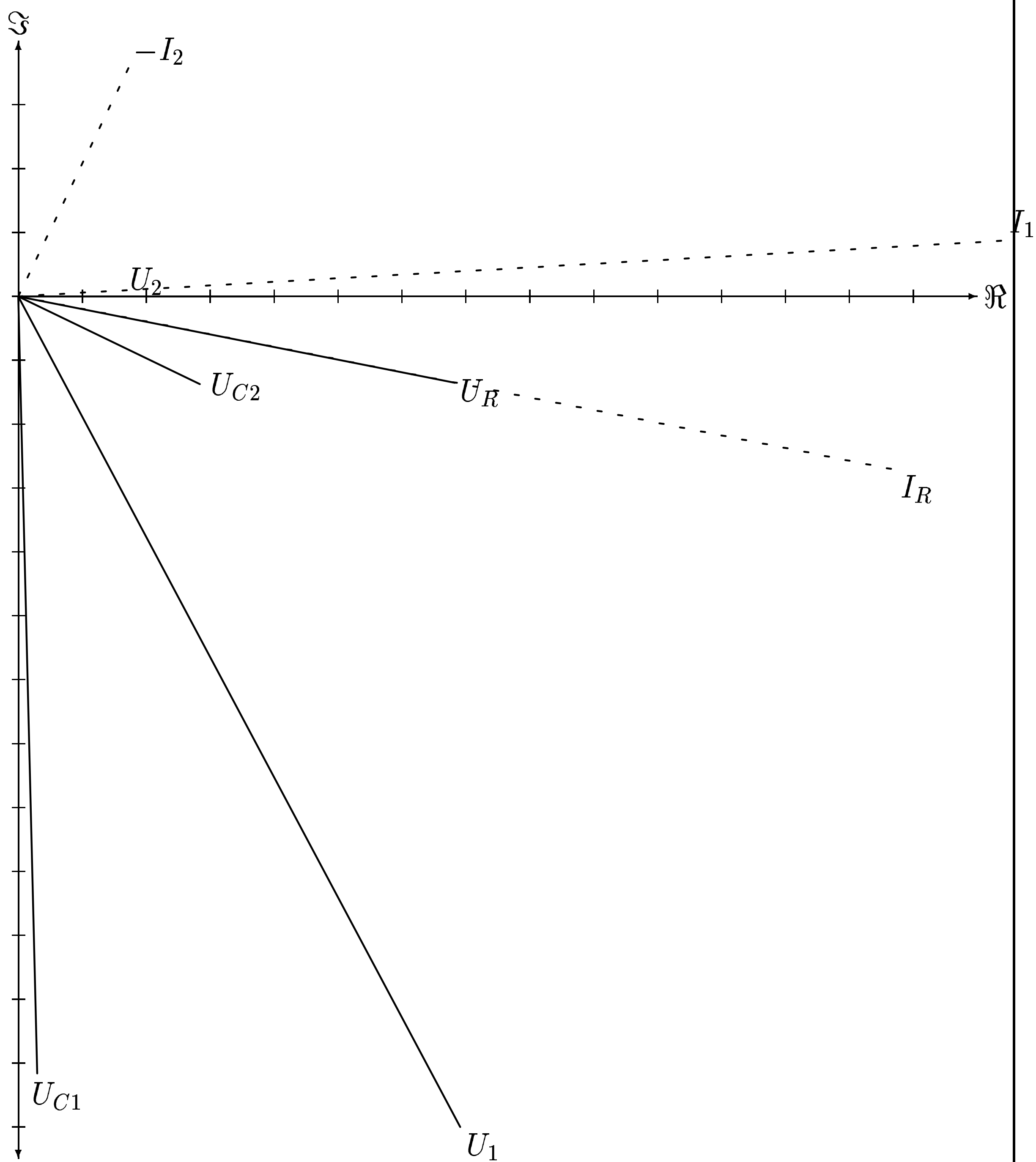
1.

$$
\begin{aligned}
Z_{1\mathrm{L}} &= Z_R + Z_C = 939,8\Omega \mathrm{e}^{-j57,86^\circ} \\
Z_{1\mathrm{K}} &= Z_C + \frac{Z_R Z_C}{Z_R + Z_C} = 1082\Omega \mathrm{e}^{-j70,65^\circ} \\
Z_W &= \sqrt{Z_{1\mathrm{K}} Z_{1\mathrm{L}}} = 1008,4\Omega \mathrm{e}^{-j64,26^\circ} \\
\tanh g_W &= \sqrt{\frac{Z_{1\mathrm{K}}}{Z_{1\mathrm{L}}}} = 1,0730\mathrm{e}^{-j6,4^\circ} = x \\
g_W &= \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) = 1,3587 - j1,0674 \\
& a_W = 1,3587\mathrm{Np} \quad b_W = -61,16^\circ
\end{aligned}
$$

2.



$$
\begin{aligned}
-I_2 &= \frac{U_2}{Z_W} = \frac{1\mathrm{V}}{1008,4\,\Omega\,\mathrm{e}^{-j64,26^\circ}} = 0.99\mathrm{mA}\,\mathrm{e}^{j64,26^\circ} \\
U_{C2} &= -I_2 Z_C = 0,7892\mathrm{V}\mathrm{e}^{-j25,74^\circ} \\
U_R &= U_{C2} + U_2 = 1,74\mathrm{V}\mathrm{e}^{-j11,2^\circ} \\
I_R &= \frac{U_R}{R} = 3,48\mathrm{mA}\mathrm{e}^{-j11,2^\circ} \\
I_1 &= I_R + (-I_2) = 3,85\mathrm{mA}\mathrm{e}^{j3,23^\circ} \\
U_{C1} &= I_1 Z_C = 3,04\mathrm{V}\mathrm{e}^{-j88,6^\circ} \\
U_1 &= U_{C1} + U_R = 3,86\mathrm{V}\mathrm{e}^{-j62^\circ}
\end{aligned}
$$



1V $\hat{=}$ 4cm, 1mA $\hat{=}$ 4cm

3.

$$g_B = \ln\left(\frac{U_0/2}{U_2}\sqrt{\frac{Z_{W_1}}{Z_{W_2}}}\right) = \ln\left(\frac{U_1}{U_2}\right) \quad \text{da } Z_{W_1} = Z_{W_2}$$

$$g_B = 1{,}3507 - j1{,}0821$$

Aufgabe 30:

.1

$$
\begin{aligned}
D_B &= \frac{1}{2}\left[A_{11}\sqrt{\frac{Z_a}{Z_i}} + \frac{A_{12}}{\sqrt{Z_i Z_a}} + A_{21}\sqrt{Z_i Z_a} + A_{22}\sqrt{\frac{Z_i}{Z_a}}\right] \\
g_B &= \ln(D_B) \\
A_{11} &= \cosh g_W \sqrt{\frac{Z_{W_1}}{Z_{W_2}}} \\
A_{12} &= \sinh g_W \sqrt{Z_{W_1} Z_{W_2}} \\
A_{21} &= \sinh g_W \sqrt{\frac{1}{Z_{W_1} Z_{W_2}}} \\
A_{22} &= \cosh g_W \sqrt{\frac{Z_{W_2}}{Z_{W_1}}} \\
g_B &= \ln\left(\frac{1}{2}\left[\cosh g_W \left(\sqrt{\frac{Z_a}{Z_i}}\sqrt{\frac{Z_{W_1}}{Z_{W_2}}} + \sqrt{\frac{Z_i}{Z_a}}\sqrt{\frac{Z_{W_2}}{Z_{W_1}}}\right) + \right.\right. \\
&\qquad \left.\left. \sinh g_W \left(\frac{\sqrt{Z_{W_1} Z_{W_2}}}{\sqrt{Z_a Z_i}} + \frac{\sqrt{Z_i Z_a}}{\sqrt{Z_{W_1} Z_{W_2}}}\right)\right]\right)
\end{aligned}
$$

Sonderfall: $Z_i = Z_a$ und $Z_{W_1} = Z_{W_2} = Z_W$:

$$g_B = \ln\left(\frac{1}{2}\left[2\cosh g_W + \left(\frac{Z_W}{Z_a} + \frac{Z_a}{Z_W}\right)\sinh g_W\right]\right)$$

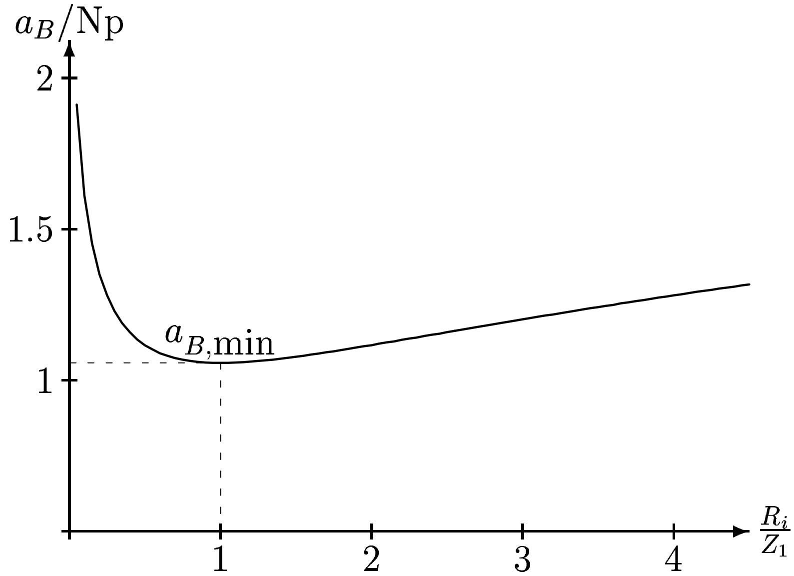
.2

$$g_B = 2,045 + j0,312$$

Aufgabe 31:

$$
\begin{aligned}
a_B &= \ln\left(\frac{U_0/2}{U_2}\sqrt{\frac{Z_a}{Z_i}}\right)\\
U_1 &= U_2\cosh g_W + I_2 Z_W \sinh g_W \quad \text{Zweitorgleichung}\\
&= U_2\left(\cosh g_W - \frac{Z_W}{R_a}\sinh g_W\right) \qquad \text{mit } -I_2 = \frac{U_2}{R_a}\\
&= U_2\left(\cosh a_W - \frac{Z_W}{R_a}\sinh a_W\right) \qquad \text{mit } g_W = a_W\,,\ b_W = 0\\
U_1 &= -0{,}80732 U_2 \quad U_2 = -1{,}2387 U_1 = kU_1\\
U_1 &= U_0\frac{Z_1}{R_i + Z_1} \quad U_2 = k\frac{Z_1}{R_i+Z_1}U_0\\
a_B &= \ln\frac{R_i+Z_1}{2kZ_1}\sqrt{\frac{Z_a}{R_i}} = \ln\sqrt{\frac{(R_i+Z_1)^2}{(2kZ_1)^2}\cdot\frac{Z_a}{R_i}}\\
a_B &= \frac{1}{2}\ln\frac{R_a(R_i+Z_1)^2}{4k^2R_iZ_1^2} = \frac{1}{2}\ln\left[\frac{R_a}{4k^2Z_1}\left(\frac{R_i}{Z_1}+2+\frac{Z_1}{R_i}\right)\right]\\
&= \frac{1}{2}\ln\frac{R_a}{4k^2Z_1} + \frac{1}{2}\ln\left(\frac{R_i}{Z_1}+2+\frac{Z_1}{R_i}\right)\\
\frac{\partial a_B}{\partial R_i} &= \frac{\partial}{\partial R_i}\frac{1}{2}\ln\left(\frac{R_i}{Z_1}+2+\frac{Z_1}{R_i}\right) \stackrel{!}{=} 0\\
&\Rightarrow R_{\mathrm{i,min}} = Z_1
\end{aligned}
$$

Minimum: $a_{\mathrm{B,min}}$ bei $R_i = Z_1$.


a_B/Np
2
1.5
1
$a_{B,\min}$
1
2
3
4
$\frac{R_i}{Z_1}$

(Die Lösungen zu den Aufgaben 32 und 33 kommen ein paar Seiten später.)

Aufgabe 34:

Es gilt :$A_{\mathrm{ges}} = A_{\mathrm{A}} A_{\mathrm{B}}$

$$\begin{aligned}
A_{\mathrm{ges}} &= \begin{pmatrix} A_{\mathrm{A},11} & A_{\mathrm{A},12} \\ A_{\mathrm{A},21} & A_{\mathrm{A},22} \end{pmatrix} \begin{pmatrix} A_{\mathrm{B},11} & A_{\mathrm{B},12} \\ A_{\mathrm{B},21} & A_{\mathrm{B},22} \end{pmatrix} \\
&= \begin{pmatrix} A_{\mathrm{A},11}A_{\mathrm{B},11} + A_{\mathrm{A},12}A_{\mathrm{B},21} & A_{\mathrm{A},11}A_{\mathrm{B},12} + A_{\mathrm{A},12}A_{\mathrm{B},22} \\ A_{\mathrm{A},21}A_{\mathrm{B},11} + A_{\mathrm{A},22}A_{\mathrm{B},21} & A_{\mathrm{A},21}A_{\mathrm{B},12} + A_{\mathrm{A},22}A_{\mathrm{B},22} \end{pmatrix} \\
&= \begin{pmatrix} \frac{Z_{\mathrm{A},11}Z_{\mathrm{B},11}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} + \frac{\det Z_{\mathrm{A}}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} & \frac{Z_{\mathrm{A},11}\det Z_{\mathrm{B}}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} + \frac{\det Z_{\mathrm{A}} Z_{\mathrm{B},22}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} \\ \frac{Z_{\mathrm{B},11}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} + \frac{Z_{\mathrm{A},22}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} & \frac{\det Z_{\mathrm{B}}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} + \frac{Z_{\mathrm{A},22}Z_{B,22}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} \end{pmatrix} \\
Z_{\mathrm{ges},11} &= \frac{A_{\mathrm{ges},11}}{A_{\mathrm{ges},21}} \\
&= \frac{\frac{Z_{\mathrm{A},11}Z_{\mathrm{B},11}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} + \frac{\det Z_{\mathrm{A}}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}}}{\frac{Z_{\mathrm{B},11}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} + \frac{Z_{\mathrm{A},22}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}}} = \frac{Z_{\mathrm{A},11}Z_{\mathrm{B},11} + \det Z_{\mathrm{A}}}{Z_{\mathrm{B},11} + Z_{\mathrm{A},22}} \\
Z_{\mathrm{ges},12} &= \frac{\det A_{\mathrm{ges}}}{A_{\mathrm{ges},21}} \\
&= \frac{1}{\frac{Z_{\mathrm{B},11}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} + \frac{Z_{\mathrm{A},22}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}}} = \frac{Z_{\mathrm{A},21}Z_{\mathrm{B},21}}{Z_{\mathrm{B},11} + Z_{\mathrm{A},22}} \\
Z_{\mathrm{ges},21} &= \frac{1}{A_{\mathrm{ges},21}} = Z_{\mathrm{ges},12} \\
Z_{\mathrm{ges},22} &= \frac{A_{\mathrm{ges},22}}{A_{\mathrm{ges},21}} = \frac{1}{A_{\mathrm{ges},21}} \\
&= \frac{\frac{\det Z_{\mathrm{B}}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} + \frac{Z_{\mathrm{A},22}Z_{\mathrm{B},22}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}}}{\frac{Z_{\mathrm{B},11}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}} + \frac{Z_{\mathrm{A},22}}{Z_{\mathrm{A},21}Z_{\mathrm{B},21}}} = \frac{\det Z_{\mathrm{B}} + Z_{\mathrm{A},22}Z_{\mathrm{B},22}}{Z_{\mathrm{B},11} + Z_{\mathrm{A},22}}
\end{aligned}$$

Aufgabe 32:

1.

$$X_D = \omega L \to L = \frac{X_D}{2\pi f_g} = 4,775\text{mH}$$

$$X_D = \sqrt{\frac{L}{C}} \to C = \frac{L}{X_D^2} = 13,263\text{nF}$$

2. Mit $\Omega = -\frac{f_g}{f}$ für den Hochpass und $\Omega = \frac{f}{f_g}$ für den Tiefpass lassen sich die folgenden Rechnungen parallel durchführen. Für die Seiten eines Halbglieds (Z_{W_1}, Z_{W_2}) gilt jeweils :

$$Z_{W_1} = X_D\sqrt{1-\Omega^2}$$

$$Z_{W_2} = X_D\frac{1}{\sqrt{1-\Omega^2}}$$

$$g_W = \ln\left(j\Omega \pm \sqrt{1-\Omega^2}\right)$$

	HP		TP	
f/kHz	10	40	10	40
Ω	-2	-0,5	0,5	2
$\sqrt{1-\Omega^2}$	$\pm j1,732$	0,866	0,866	$\pm j1,732$
Z_{W_1}/Ω	-j1039,2	519,6	519,6	j1039,2
$\frac{1}{\sqrt{1-\Omega^2}}$	$\pm j0,577$	1,155	1,155	$\pm j0,577$
Z_{W_2}/Ω	$j346,4$	692,8	692,8	$-j346,4$

$$g_W = \ln\left(j\Omega \pm \sqrt{1-\Omega^2}\right)$$

HP, 10kHz, $\Omega = -2,0$ $\quad g_W = \ln\left(-j2 - \sqrt{1-(-2)^2}\right) = 1,3169 - j\frac{\pi}{2}$

HP, 40kHz, $\Omega = -0,5$ $\quad g_W = \ln\left(-j0,5 + \sqrt{1-(-0,5)^2}\right) = -j\frac{\pi}{6}$

TP, 10kHz, $\Omega = +0,5 \qquad g_W = \ln\left(j0,5 + \sqrt{1-(0,5)^2}\right) = j\frac{\pi}{6}$

TP, 40kHz, $\Omega = +2,0 \qquad g_W = \ln\left(j2 - \sqrt{1-(2)^2}\right) = 1,3169 + j\frac{\pi}{2}$

3.

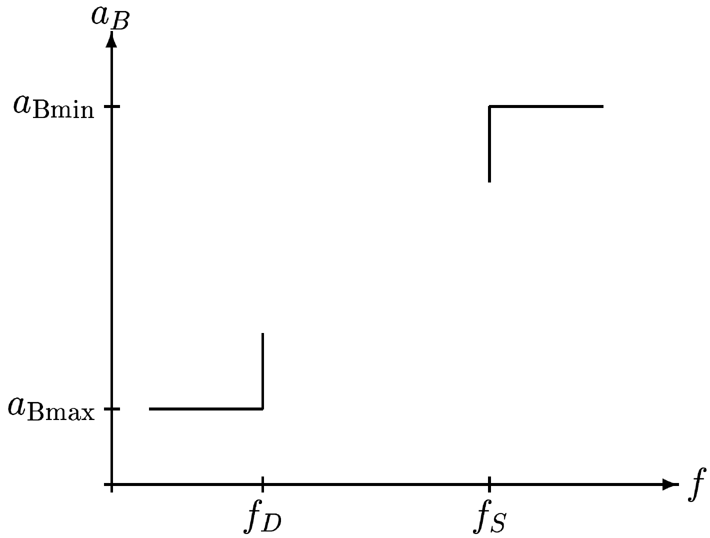
$$g_B = \ln\frac{1}{2}\left[\cosh g_W \left(\sqrt{\frac{Z_a}{Z_i}}\sqrt{\frac{Z_{W_1}}{Z_{W_2}}} + \sqrt{\frac{Z_i}{Z_a}}\sqrt{\frac{Z_{W_2}}{Z_{W_1}}}\right) + \sinh g_W \left(\frac{\sqrt{Z_{W_1}Z_{W_2}}}{\sqrt{Z_a Z_i}} + \frac{\sqrt{Z_i Z_a}}{\sqrt{Z_{W_1}Z_{W_2}}}\right)\right]$$

$$g_W = -j\pi/6 \quad , \quad Z_{W_1} = 519,6\Omega \quad , \quad Z_{W_2} = 692,8\Omega$$

$$g_B = 0,00236 - j0,52224$$

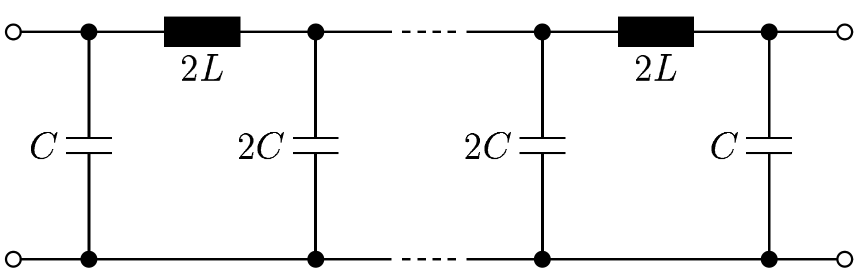
Aufgabe 33:

Toleranzschema



Dimensionierung für ein Grundglied

$$\begin{aligned}
a_{\text{Bmax}} &= 0,3\text{Np} \\
Q &= \mathrm{e}^{a_{\text{Bmax}}} + \sqrt{\mathrm{e}^{2a_{\text{Bmax}}} - 1} = 2,2565 \\
\Omega_D &= \sqrt{1 - \frac{1}{Q^4}} = 0,9805 \\
f_g &= \frac{f_D}{\Omega_D} = 6,527\text{kHz} \\
\Omega_S &= \frac{f_S}{f_g} = 1,2256 \\
n &\geq \frac{arcosh\left(e^{a_{\text{Bmin}}}\right)}{2arcosh|\Omega_s|} = 6,59 \\
\Rightarrow n &= 7 \Rightarrow \text{ 7-gliedrige Kette}
\end{aligned}$$



Bestimmung der Bauelemente:

$$Q = \frac{Z_a}{X_D} \;\Rightarrow\; X_D = 531,80\Omega$$

Obige Formel ist gültig für π-Grundglied !

$$X_D = 2\pi f_g L \;\Rightarrow\; L = \frac{X_D}{2\pi f_g} = 12,967\text{mH}$$

$$X_D = \sqrt{\frac{L}{C}} \;\Rightarrow\; C = \frac{L}{X_D^2} = 45,852\text{nF}$$

(Die Lösung zu Aufgabe 34 findet sich ein paar Seiten vorher.)

Aufgabe 35:

.1

$$Z_{1\mathrm{K}} = \left.\frac{U_1}{I_1}\right|_{U_2=0} = j\omega L + \frac{1}{j\omega C} = j\left(\omega L - \frac{1}{\omega C}\right) = j\frac{\omega^2 LC - 1}{\omega C}$$

$$\begin{aligned}
Z_{1\mathrm{L}} &= \left.\frac{U_1}{I_1}\right|_{I_2=0} = j\omega L + \frac{1}{j\omega C} + \frac{1}{\frac{1}{j\omega L} + j\omega C} = j\frac{\omega^2 LC - 1}{\omega C} + j\frac{\omega L}{1-\omega^2 LC} \\
&= j\frac{(\omega^2 LC - 1)(1-\omega^2 LC) + \omega^2 LC}{\omega C(1-\omega^2 LC)} \\
&= j\frac{\omega^2 LC - 1 - \omega^4 L^2 C^2 + \omega^2 LC + \omega^2 LC}{\omega C(1-\omega^2 LC)} \\
&= j\frac{3\omega^2 LC - \omega^4 L^2 C^2 - 1}{\omega C(1-\omega^2 LC)} = j\frac{1 - 3\omega^2 LC + \omega^4 L^2 C^2}{\omega C(\omega^2 LC - 1)}
\end{aligned}$$

$$\begin{aligned}
Z_{2\mathrm{K}} &= \left.\frac{U_2}{I_2}\right|_{U_1=0} = \frac{1}{\frac{1}{j\omega L + \frac{1}{j\omega C}} + \frac{1}{j\omega L} + j\omega C} = \frac{1}{\frac{j\omega C}{1-\omega^2 LC} + \frac{1-\omega^2 LC}{j\omega L}} \\
&= j\frac{\omega L(1-\omega^2 LC)}{(1-\omega^2 LC)^2 - \omega^2 LC} = j\frac{\omega L(1-\omega^2 LC)}{1 - 3\omega^2 LC + \omega^4 L^2 C^2}
\end{aligned}$$

$$Z_{2\mathrm{L}} = \left.\frac{U_2}{I_2}\right|_{I_1=0} = \frac{1}{\frac{1}{j\omega L} + j\omega C} = j\frac{\omega L}{1-\omega^2 LC}$$

.2

$$\begin{aligned}
Z_{\mathrm{W1}} &= \sqrt{Z_{1\mathrm{K}} Z_{1\mathrm{L}}} = \sqrt{j\frac{\omega^2 LC - 1}{\omega C} j\frac{1 - 3\omega^2 LC + \omega^4 L^2 C^2}{\omega C(\omega^2 LC - 1)}} \\
&= \sqrt{\frac{3\omega^2 LC - \omega^4 L^2 C^2 - 1}{\omega^2 C^2}} \\
&= \frac{1}{\omega C}\sqrt{3\omega^2 LC - \omega^4 L^2 C^2 - 1}
\end{aligned}$$

$$Z_{\mathrm{W2}} = \sqrt{Z_{2\mathrm{K}} Z_{2\mathrm{L}}} = \sqrt{j\frac{\omega L(1-\omega^2 LC)}{1 - 3\omega^2 LC + \omega^4 L^2 C^2} j\frac{\omega L}{1-\omega^2 LC}}$$

$$= \sqrt{\frac{\omega^2 L^2}{-1 + 3\omega^2 LC - \omega^4 L^2 C^2}}$$

$$= \frac{\omega L}{\sqrt{3\omega^2 LC - \omega^4 L^2 C^2 - 1}}$$

.3

$$\tanh g_W = \sqrt{\frac{Z_{1\mathrm{K}}}{Z_{1\mathrm{L}}}} = \sqrt{\frac{j\frac{\omega^2 LC - 1}{\omega C}}{j\frac{1 - 3\omega^2 LC + \omega^4 L^2 C^2}{\omega C(\omega^2 LC - 1)}}}$$

$$= \sqrt{\frac{(\omega^2 LC - 1)^2}{1 - 3\omega^2 LC + \omega^4 L^2 C^2}} = \frac{\omega^2 LC - 1}{\sqrt{1 - 3\omega^2 LC + \omega^4 L^2 C^2}}$$

$$g_W = \operatorname{artanh}\left(\frac{\omega^2 LC - 1}{\sqrt{1 - 3\omega^2 LC + \omega^4 L^2 C^2}}\right)$$

Aufgabe 36:

1. Bandpaß

$$H_{\mathrm{BP}}(\omega) = \frac{U_2}{U_1} = \frac{Z_p}{Z_r + Z_p}$$

mit $$Z_r = j\omega L_r + \frac{1}{j\omega C_r} = j\frac{\omega^2 L_r C_r - 1}{\omega C_r}$$

und $$Z_p = \frac{1}{\frac{1}{j\omega L_p} + j\omega C_p} = j\frac{\omega L_p}{1 - \omega^2 L_p C_p}$$

$$\begin{aligned} H_{\mathrm{BP}}(\omega) &= \frac{\frac{\omega L_p}{1-\omega^2 L_p C_p}}{\frac{\omega^2 L_r C_r - 1}{\omega C_r} + \frac{\omega L_p}{1-\omega^2 L_p C_p}} \\ &= \frac{\omega L_p}{1-\omega^2 L_p C_p} \cdot \frac{\omega C_r (1-\omega^2 L_p C_p)}{(\omega^2 L_r C_r - 1)(1-\omega^2 L_p C_p) + \omega^2 L_p C_r} \\ &= \frac{\omega^2 L_p C_r}{\omega^2 (L_r C_r + L_p C_p + L_p C_r) - \omega^4 L_p C_p L_r C_r - 1} \end{aligned}$$

Bandsperre

$$\begin{aligned} H_{\mathrm{BS}}(\omega) &= \frac{U_2}{U_1} = \frac{Z_r}{Z_r + Z_p} \\ &= \frac{\frac{\omega^2 L_r C_r - 1}{\omega C_r}}{\frac{\omega^2 L_r C_r - 1}{\omega C_r} + \frac{\omega L_p}{1-\omega^2 L_p C_p}} \\ &= \frac{\omega^2 L_r C_r - 1}{\omega C_r} \cdot \frac{\omega C_r (1-\omega^2 L_p C_p)}{(\omega^2 L_r C_r - 1)(1-\omega^2 L_p C_p) + \omega^2 L_p C_r} \\ &= \frac{(\omega^2 L_r C_r - 1)(1-\omega^2 L_p C_p)}{(\omega^2 L_r C_r - 1)(1-\omega^2 L_p C_p) + \omega^2 L_p C_r} \\ &= \frac{\omega^2 (L_r C_r + L_p C_p) - \omega^4 L_p C_p L_r C_r - 1}{\omega^2 (L_r C_r + L_p C_p + L_p C_r) - \omega^4 L_p C_p L_r C_r - 1} \end{aligned}$$

2. Grenzfrequenzen $|H(\omega)| = \frac{1}{\sqrt{2}}$

Bandpaß

$$\left|\frac{\omega^2 L_p C_r}{\omega^2(L_r C_r + L_p C_p + L_p C_r) - \omega^4 L_p C_p L_r C_r - 1}\right| = \frac{1}{\sqrt{2}}$$

$$\left|\omega^2(L_r C_r + L_p C_p + L_p C_r) - \omega^4 L_p C_p L_r C_r - 1\right| = \sqrt{2}\omega^2 L_p C_r$$

$$\omega^2(L_r C_r + L_p C_p + L_p C_r) - \omega^4 L_p C_p L_r C_r - 1 = \pm\sqrt{2}\omega^2 L_p C_r$$

$$\omega^4 L_p C_p L_r C_r - \omega^2(L_r C_r + L_p C_p + (1 \pm \sqrt{2}) L_p C_r) + 1 = 0$$

$$\omega_{g1,2}^2 = \frac{L_r C_r + L_p C_p + (1 \pm \sqrt{2}) L_p C_r}{2 L_p C_p L_r C_r} \pm \sqrt{\left(\frac{L_r C_r + L_p C_p + (1 \pm \sqrt{2}) L_p C_r}{2 L_p C_p L_r C_r}\right)^2 - \frac{1}{L_p C_p L_r C_r}}$$

Vorzeichenwahl für $\sqrt{2}$ so, daß die Diskriminante größer 0 wird.
Bandsperre

$$\left|\frac{\omega^2(L_r C_r + L_p C_p) - \omega^4 L_p C_p L_r C_r - 1}{\omega^2(L_r C_r + L_p C_p + L_p C_r) - \omega^4 L_p C_p L_r C_r - 1}\right| = \frac{1}{\sqrt{2}}$$

$$\pm\sqrt{2}\omega^2(L_r C_r + L_p C_p) \mp \sqrt{2}\omega^4 L_p C_p L_r C_r \mp \sqrt{2} = \omega^2(L_r C_r + L_p C_p + L_p C_r) - \omega^4 L_p C_p L_r C_r - 1$$

$$\omega^4 L_p C_p L_r C_r (1 \mp \sqrt{2}) + \omega^2[(\pm\sqrt{2} - 1)(L_r C_r + L_p C_p) - L_p C_r] + 1 \mp \sqrt{2} = 0$$

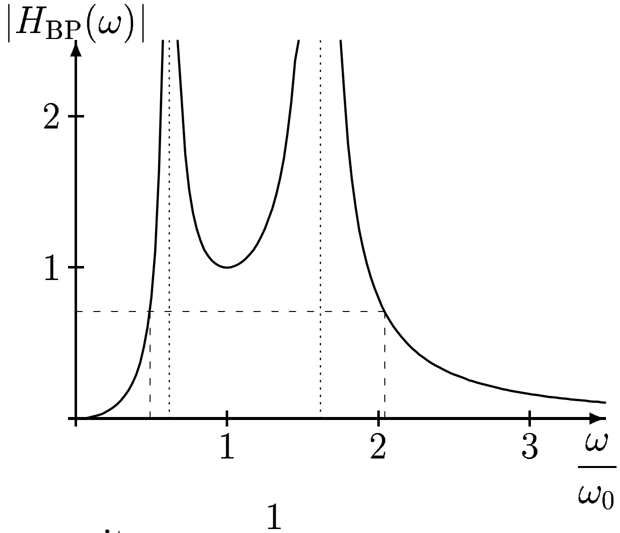
$$\omega^4 + \omega^2 \frac{(\pm\sqrt{2} - 1)(L_r C_r + L_p C_p) - L_p C_r}{(1 \mp \sqrt{2}) L_p C_p L_r C_r} + \frac{1}{L_p C_p L_r C_r} = 0$$

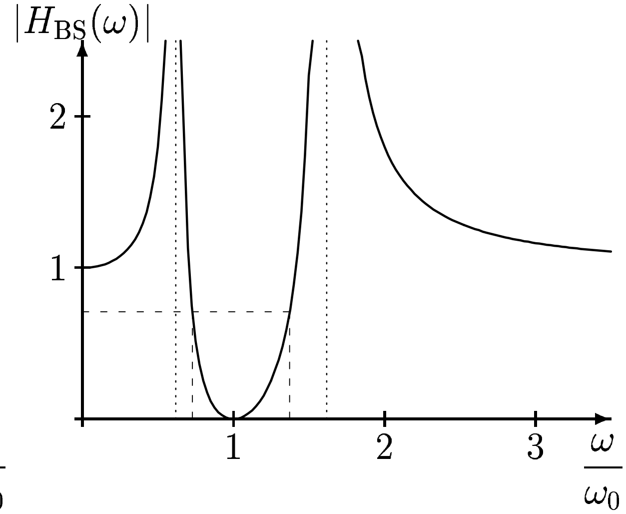
$$\omega_{g1,2}^2 = -\frac{(\pm\sqrt{2} - 1)(L_r C_r + L_p C_p) - L_p C_r}{2(1 \mp \sqrt{2}) L_p C_p L_r C_r} \pm \sqrt{\left(\frac{(\pm\sqrt{2} - 1)(L_r C_r + L_p C_p) - L_p C_r}{2(1 \mp \sqrt{2}) L_p C_p L_r C_r}\right)^2 - \frac{1}{L_p C_p L_r C_r}}$$

Vorzeichenwahl für $\sqrt{2}$ so, daß die Diskriminante größer 0 wird.

3. Spezialfall: $L_p = L_r = L$, $C_p = C_r = C$

$$H_{\mathrm{BP}}(\omega) = \frac{\omega^2 LC}{3\omega^2 LC - \omega^4 L^2 C^2 - 1} \qquad H_{\mathrm{BS}}(\omega) = \frac{2\omega^2 LC - \omega^4 L^2 C^2 - 1}{3\omega^2 LC - \omega^4 L^2 C^2 - 1}$$





mit $\omega_0 = \frac{1}{\sqrt{LC}}$

Polstellen identisch für BP und BS:

$$\begin{aligned} 3\omega^2 LC - \omega^4 L^2 C^2 - 1 &= 0 \\ \omega^4 - \frac{3\omega^2}{LC} + \frac{1}{L^2C^2} &= 0 \end{aligned}$$

$$\omega^2_{\infty 1,2} = \frac{3}{2LC} \pm \sqrt{\frac{9}{4L^2C^2} - \frac{1}{L^2C^2}} = \frac{1}{LC}\left(\frac{3}{2} \pm \frac{\sqrt{5}}{2}\right) = \frac{3 \pm \sqrt{5}}{2LC}$$

$$\omega_{\infty 1,2} = \sqrt{\frac{3 \pm \sqrt{5}}{2LC}}$$

$$\omega_{\infty 1} = \sqrt{\frac{3 - \sqrt{5}}{2LC}} = \frac{0,6180}{\sqrt{LC}}, \quad \omega_{\infty 2} = \sqrt{\frac{3 + \sqrt{5}}{2LC}} = \frac{1,6180}{\sqrt{LC}}$$

Grenzfrequenzen: Bandpaß

$$\omega^2_{g1,2} = \frac{3 \pm \sqrt{2}}{2LC} \pm \sqrt{\left(\frac{3 \pm \sqrt{2}}{2LC}\right)^2 - \frac{1}{L^2C^2}}, \quad \left(\frac{3 + \sqrt{2}}{2}\right)^2 - 1 = \frac{7 + 6\sqrt{2}}{4} > 0$$

$$= \frac{3 + \sqrt{2} \pm \sqrt{7 + 6\sqrt{2}}}{2LC}$$

$$\omega_{g1,2} = \sqrt{\frac{3+\sqrt{2}\pm\sqrt{7+6\sqrt{2}}}{2LC}} = \frac{1}{\sqrt{LC}}\sqrt{\frac{3+\sqrt{2}\pm\sqrt{7+6\sqrt{2}}}{2}}$$

$$= \frac{1}{\sqrt{LC}}\sqrt{\frac{4,4142\pm 3,9351}{2}}$$

$$\omega_{g1} = \frac{0,4894}{\sqrt{LC}} \qquad \omega_{g2} = \frac{2,0432}{\sqrt{LC}}$$

Bandsperre

$$\omega_{g1,2}^2 = \frac{(\pm\sqrt{2}-1)2-1}{2(1\mp\sqrt{2})LC} \pm \frac{1}{LC}\sqrt{\left(\frac{(\pm\sqrt{2}-1)2-1}{2(1\mp\sqrt{2})}\right)^2 - 1}$$

$$\left(\frac{2(\sqrt{2}-1)-1}{2(1-\sqrt{2})}\right)^2 - 1 = \frac{5-4\sqrt{2}}{12-8\sqrt{2}} = -0,9571 < 0$$

$$\omega_{g1,2}^2 = -\frac{(-\sqrt{2}-1)2-1}{2(1+\sqrt{2})LC} \pm \frac{1}{LC}\sqrt{\left(\frac{(-\sqrt{2}-1)2-1}{2(1+\sqrt{2})}\right)^2 - 1}$$

$$= \frac{1}{LC}\left(\frac{3+2\sqrt{2}}{2+2\sqrt{2}} \pm \sqrt{\frac{5+4\sqrt{2}}{12+8\sqrt{2}}}\right) = \frac{1}{LC}(1,2071 \pm 0,6761)$$

$$\omega_{g1,2} = \frac{1}{\sqrt{LC}}\sqrt{1,2071\pm 0,6761}$$

$$\omega_{g,1} = \frac{0,7287}{\sqrt{LC}} \qquad \omega_{g,2} = \frac{1,3723}{\sqrt{LC}}$$

Aufgabe 37:

Dualität: $L_pC_p = L_rC_r$

Dualitätskonstante: $X_D = \sqrt{\frac{L_r}{C_p}} = \sqrt{\frac{L_p}{C_r}}$

Resonanzfrequenz: $\omega_0 = \frac{1}{\sqrt{LC}}$

Bandpaß:

$$\begin{aligned}
Z_{W_1} &= \sqrt{Z_{1\mathrm{K}} Z_{1\mathrm{L}}} \\
Z_{1\mathrm{K}} &= j\omega L_r + \frac{1}{j\omega C_r} = j\frac{\omega^2 L_r C_r - 1}{\omega C_r} \\
Z_{1\mathrm{L}} &= Z_{1\mathrm{K}} + \frac{1}{j\omega C_p + \frac{1}{j\omega L_p}} = j\frac{\omega^2 L_r C_r - 1}{\omega C_r} + j\frac{\omega L_p}{1-\omega^2 L_p C_p} \\
&= j\frac{\omega^2 L_p C_r - (1-\omega^2 L_p C_p)^2}{(1-\omega^2 L_p C_p)\omega C_r} \\
Z_{W_1} &= \sqrt{j\frac{\omega^2 L_r C_r - 1}{\omega C_r} j\frac{\omega^2 L_p C_r - (1-\omega^2 L_p C_p)^2}{(1-\omega^2 L_p C_p)\omega C_r}} \\
&= \sqrt{\frac{\omega^2 L_p C_r - (1-\omega^2 L_p C_p)^2}{\omega^2 C_r^2}} = \sqrt{\frac{L_p}{C_r}}\sqrt{1 - \frac{1}{L_p C_r}\frac{(1-\omega^2 L_p C_p)^2}{\omega^2}} \\
&= X_D\sqrt{1-\Omega_{\mathrm{BP}}^2} \\
\Omega_{\mathrm{BP}} &= \frac{1}{\sqrt{L_p C_r}}\frac{1-\omega^2 L_p C_p}{\omega} = \frac{1}{\sqrt{L_p C_r}}\left(\frac{1}{\omega} - \frac{\omega}{\omega_0^2}\right) = \frac{1}{\sqrt{L_p C_r}}\frac{1}{\omega_0}\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right) \\
&= \frac{1}{\sqrt{L_p C_r}}\sqrt{L_p C_p}\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right) = \sqrt{\frac{C_p}{C_r}}\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)
\end{aligned}$$

Die übliche Darstellung ist

$$\Omega_{\mathrm{BP}} = \sqrt{\frac{C_p}{C_r}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

Umrechnung von Ω_{BP} nach ω:

$$\sqrt{\frac{C_r}{C_p}}\Omega = \frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}$$

$$\omega\omega_0\sqrt{\frac{C_r}{C_p}}\Omega = \omega_0^2 - \omega^2$$

$$\omega^2 + \omega\omega_0\sqrt{\frac{C_r}{C_p}}\Omega - \omega_0^2 = 0$$

$$\omega_{1,2} = -\frac{\omega_0}{2}\sqrt{\frac{C_r}{C_p}}\Omega \pm \sqrt{\frac{\omega_0^2 C_r}{4C_p}\Omega^2 + \omega_0^2}$$

$$\omega = \frac{\omega_0}{2}\left(-\sqrt{\frac{C_r}{C_p}}\Omega + \sqrt{\frac{C_r}{C_p}\Omega^2 + 4}\right)$$

Wellenwiderstand Z_{W_2}:

$$Z_{W_2} = \sqrt{Z_{2\mathrm{K}} Z_{2\mathrm{L}}}$$

$$Z_{2\mathrm{K}} = \frac{1}{\frac{1}{j\omega L_r + \frac{1}{j\omega C_r}} + \frac{1}{j\omega L_p} + j\omega C_p}$$

$$= \frac{1}{\frac{j\omega C_r}{1-\omega^2 L_r C_r} + \frac{1-\omega^2 L_p C_p}{j\omega L_p}} = j\frac{1}{\frac{-\omega^2 L_p C_r + (1-\omega^2 L_p C_p)^2}{(1-\omega^2 L_r C_r)\omega L_p}}$$

$$Z_{2\mathrm{L}} = \frac{1}{\frac{1}{j\omega L_p} + j\omega C_p} = j\frac{1}{\frac{1-\omega^2 L_p C_p}{\omega L_p}}$$

$$Z_{W_2} = \sqrt{j\frac{1}{\frac{-\omega^2 L_p C_r + (1-\omega^2 L_p C_p)^2}{(1-\omega^2 L_r C_r)\omega L_p}} j\frac{1}{\frac{1-\omega^2 L_p C_p}{\omega L_p}}}$$

$$= \frac{1}{\sqrt{\frac{\omega^2 L_p C_r - (1-\omega^2 L_p C_p)^2}{\omega^2 L_p^2}}} = \frac{1}{\sqrt{\frac{C_r}{L_p}}\sqrt{1 - \frac{1}{L_p C_r}\frac{(1-\omega^2 L_p C_p)^2}{\omega^2}}}$$

$$= \frac{X_D}{\sqrt{1-\Omega_{\mathrm{BP}}^2}}$$

Bandsperre:

$$Z_{W_1} = \sqrt{Z_{1\mathrm{K}} Z_{1\mathrm{L}}}$$

$$
\begin{aligned}
Z_{1\text{K}} &= \frac{1}{\frac{1}{j\omega L_p} + j\omega C_p} = j\frac{\omega L_p}{1-\omega^2 L_p C_p} \\
Z_{1\text{L}} &= Z_{1\text{K}} + j\omega L_r + \frac{1}{j\omega C_r} = j\frac{\omega L_p}{1-\omega^2 L_p C_p} + j\frac{\omega^2 L_r C_r - 1}{\omega C_r} \\
&= j\frac{\omega^2 L_p C_r + (\omega^2 L_r C_r - 1)(1-\omega^2 L_p C_p)}{(1-\omega^2 L_p C_p)\omega C_r} \qquad L_p C_p = L_r C_r \\
&= j\frac{\omega^2 L_p C_r - (1-\omega^2 L_p C_p)^2}{(1-\omega^2 L_p C_p)\omega C_r} \\
Z_{W_1} &= \sqrt{j\frac{\omega L_p}{1-\omega^2 L_p C_p} j\frac{\omega^2 L_p C_r - (1-\omega^2 L_p C_p)^2}{(1-\omega^2 L_p C_p)\omega C_r}} \\
&= \sqrt{\frac{L_p}{C_r}}\sqrt{\frac{(1-\omega^2 L_p C_p)^2 - \omega^2 L_p C_r}{(1-\omega^2 L_p C_p)^2}} \\
&= \sqrt{\frac{L_p}{C_r}}\sqrt{1 - \frac{\omega^2 L_p C_r}{(1-\omega^2 L_p C_p)^2}} = X_D\sqrt{1-\Omega_{\text{BS}}^2} \\
\Omega_{\text{BS}} &= \sqrt{\frac{\omega^2 L_p C_r}{(1-\omega^2 L_p C_p)^2}} = \frac{\omega\sqrt{L_p C_r}}{1-\omega^2 L_p C_p} = \sqrt{L_p C_r}\cdot\frac{\omega}{1-\frac{\omega^2}{\omega_0^2}}, \quad \omega_0 = \frac{1}{\sqrt{L_p C_p}} \\
&= -\sqrt{L_p C_r}\cdot\frac{\omega_0}{\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}} = \frac{\sqrt{L_p C_r}}{\sqrt{L_p C_p}}\cdot\frac{-1}{\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}} = \sqrt{\frac{C_r}{C_p}}\cdot\frac{1}{\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}} = -\frac{1}{\Omega_{\text{BP}}}
\end{aligned}
$$

Die übliche Darstellung ist

$$
\Omega_{\text{BS}} = \frac{-1}{\sqrt{\frac{C_p}{C_r}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}
$$

Umrechnung von Ω_{BS} nach ω:

$$
\begin{aligned}
\sqrt{\frac{C_r}{C_p}}\frac{1}{\Omega} &= \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \\
\omega\omega_0\frac{1}{\Omega}\sqrt{\frac{C_r}{C_p}} &= \omega^2 - \omega_0^2 \\
\omega^2 - \frac{\omega_0}{\Omega}\sqrt{\frac{C_r}{C_p}}\omega - \omega_0^2 &= 0
\end{aligned}
$$

$$\omega_{1,2} = \frac{\omega_0}{2\Omega}\sqrt{\frac{C_r}{C_p}} \pm \sqrt{\frac{\omega_0^2}{4\Omega^2}\frac{C_r}{C_p} + \omega_0^2}$$

$$\omega = \omega_0\left(\frac{1}{2\Omega}\sqrt{\frac{C_r}{C_p}} + \sqrt{\frac{1}{4\Omega^2}\frac{C_r}{C_p} + 1}\right)$$

Wellenwiderstand Z_{W_2}:

$$Z_{W_2} = \sqrt{Z_{2\mathrm{K}} Z_{2\mathrm{L}}}$$

$$Z_{2\mathrm{K}} = \frac{1}{\frac{1}{j\omega L_p} + j\omega C_p + \frac{1}{j\omega L_r + \frac{1}{j\omega C_r}}} = Z_{2\mathrm{K}}^{\mathrm{BP}}$$

$$= j\frac{(1-\omega^2 L_r C_r)\omega L_p}{-\omega^2 L_p C_r + (1-\omega^2 L_p C_p)^2}$$

$$Z_{2\mathrm{L}} = j\omega L_r + \frac{1}{j\omega C_r} = j\frac{\omega^2 L_r C_r - 1}{\omega C_r} = Z_{1\mathrm{K}}^{\mathrm{BP}}$$

$$Z_{W_2} = \sqrt{j\frac{(1-\omega^2 L_r C_r)\omega L_p}{-\omega^2 L_p C_r + (1-\omega^2 L_p C_p)^2} j\frac{\omega^2 L_r C_r - 1}{\omega C_r}}$$

$$= \sqrt{\frac{L_p}{C_r}}\sqrt{\frac{(1-\omega^2 L_p C_p)^2}{(1-\omega^2 L_p C_p)^2 - \omega^2 L_p C_r}} = \frac{X_D}{\sqrt{\frac{(1-\omega^2 L_p C_p)^2 - \omega^2 L_p C_r}{(1-\omega^2 L_p C_p)^2}}}$$

$$= \frac{X_D}{\sqrt{1 - \frac{\omega^2 L_p C_r}{(1-\omega^2 L_p C_p)^2}}} = \frac{X_D}{\sqrt{1-\Omega_{\mathrm{BS}}^2}}$$

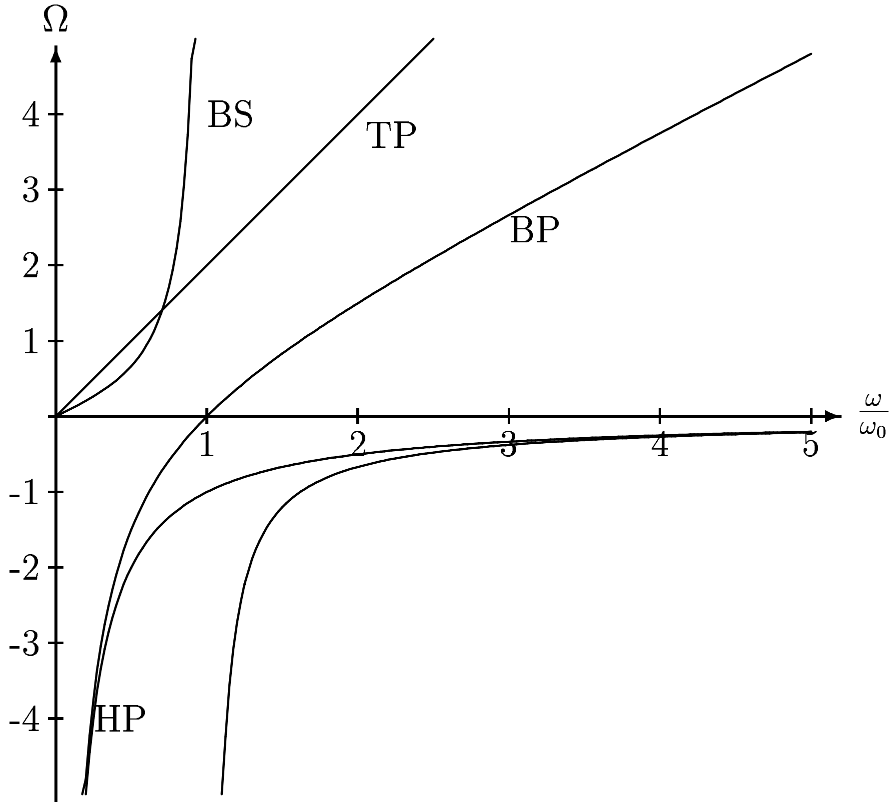
Zusammenfassung:

$$\text{TP: } \Omega_{\mathrm{TP}} = \frac{\omega}{\omega_0} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\text{HP: } \Omega_{\mathrm{HP}} = -\frac{\omega_0}{\omega} = -\frac{1}{\Omega_{\mathrm{TP}}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\text{BP: } \Omega_{\mathrm{BP}} = \sqrt{\frac{C_p}{C_r}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \qquad \omega_0 = \frac{1}{\sqrt{L_p C_p}} = \frac{1}{\sqrt{L_r C_r}}$$

$$\text{BS: } \Omega_{\text{BS}} = \frac{-1}{\sqrt{\frac{C_p}{C_r}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} = -\frac{1}{\Omega_{\text{BP}}} \qquad \omega_0 = \frac{1}{\sqrt{L_p C_p}} = \frac{1}{\sqrt{L_r C_r}}$$



Aufgabe 38:

1. Grundglied und Halbglied haben dieselben Grenzfrequenzen. Für das Halbglied gilt:

$$f_0 = \sqrt{f_- f_+} \quad \rightarrow \quad f_0 = 6\text{kHz}$$

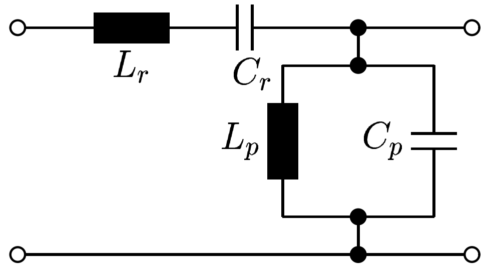
$$2\pi\Delta f = \frac{X_D}{L_r} \quad \rightarrow \quad L_r = \frac{X_D}{2\pi\Delta f} = 11,11\text{mH}$$

$$X_D = \sqrt{\frac{L_r}{C_p}} \quad \rightarrow \quad C_p = \frac{L_r}{X_D^2} = 28,146\text{nF}$$

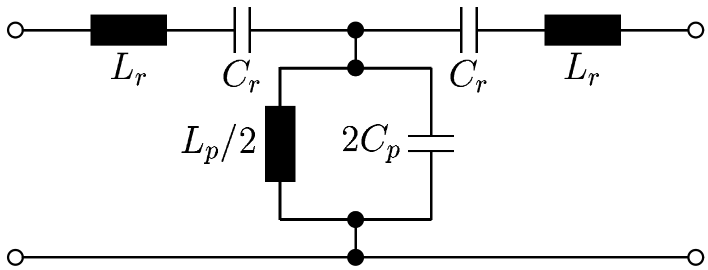
$$2\pi f_0 = \frac{1}{\sqrt{L_p C_p}} \quad \rightarrow \quad L_p = \frac{1}{4\pi^2 f_0^2 C_p} = 24,999\text{mH}$$

$$\frac{C_r}{C_p} = \frac{L_p}{L_r} \quad \rightarrow \quad C_r = \frac{C_p L_p}{L_r} = 63,326\text{nF}$$

Aufbau des Halbglieds

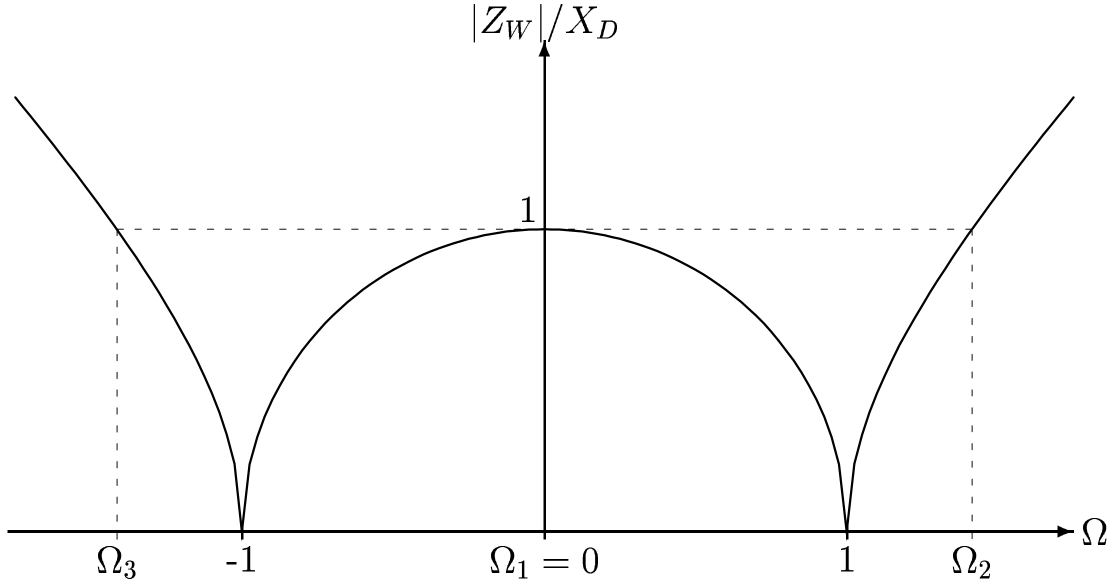


Grundglied besteht aus 2 Halbgliedern



2.

$$
\begin{aligned}
Z_W &= X_D\sqrt{1-\Omega^2} \\
|Z_W| &= X_D \Rightarrow |\sqrt{1-\Omega^2}| = 1 \\
\sqrt{1-\Omega_1^2} &= 1 \Rightarrow \Omega_1 = 0 \quad Z_W(\Omega_1) = X_D \\
\sqrt{1-\Omega_{2,3}^2} &= \pm j \Rightarrow \Omega_{2,3} = \pm\sqrt{2} \quad Z_W(\Omega_{2,3}) = \pm j X_D
\end{aligned}
$$



Umrechnung von Ω in ω und f

$$\begin{aligned} \omega &= \omega_0 \frac{1}{2}\left[\Omega\sqrt{\frac{L_p}{L_r}} + \sqrt{\Omega^2 \frac{L_p}{L_r} + 4}\right] \\ f_1 &= 6\text{kHz} \quad f_2 = 15,11\text{kHz} \quad f_3 = 2,382\text{kHz} \end{aligned}$$

Aufgabe 39:

1. Dimensionierung des Halbglieds

Anpassung: $\frac{R}{X_D} = 1,25 \Rightarrow X_D = 800\Omega$

Bandbreite: $\Delta\omega = 2\pi\Delta f = \frac{X_D}{L_r} \Rightarrow L_r = \frac{X_D}{\Delta\omega} = 31,831\text{mH}$

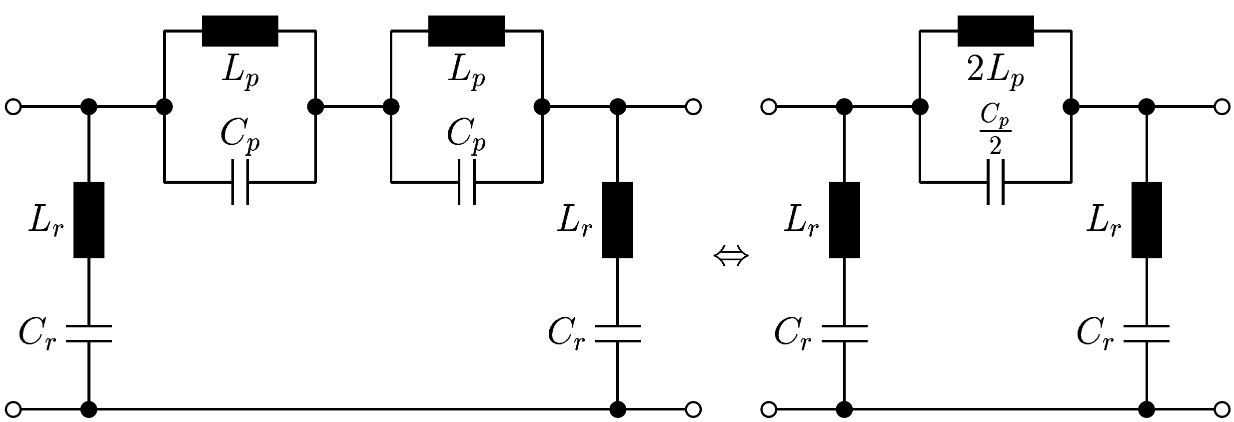
Dualität: $X_D = \sqrt{\frac{L_r}{C_p}} \Rightarrow C_p = \frac{L_r}{X_D^2} = 49,736\text{nF}$

Bandmitte: $f_0 = \sqrt{f_- f_+} \Rightarrow f_0 = \sqrt{32}\text{kHz}$

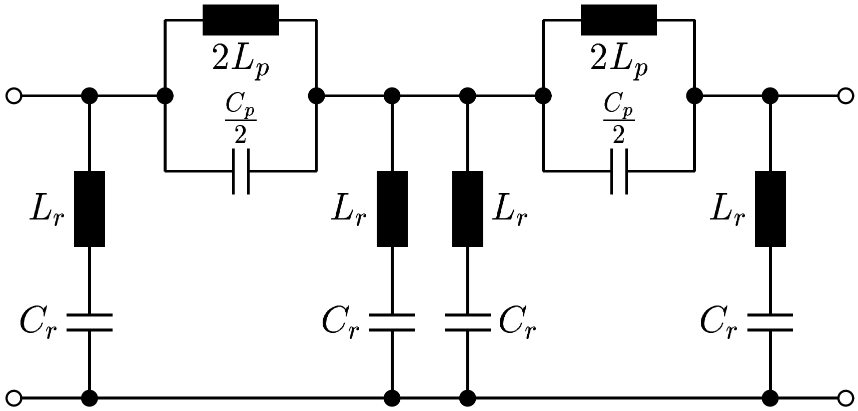
Resonanzfrequenz: $f_0 = \frac{1}{2\pi\sqrt{L_p C_p}} \Rightarrow L_p = \frac{1}{4\pi^2 f_o^2 C_p} = 15,915\text{mH}$

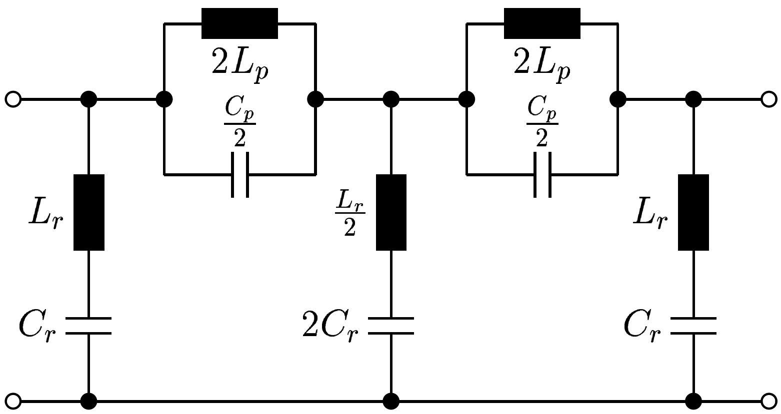
Dualität: $\frac{C_r}{C_p} = \frac{L_p}{L_r} \Rightarrow C_r = \frac{C_p L_p}{L_r} = 24,867\text{nF}$

Π-Schaltung aus zwei Halbgliedern



Bandsperre-Kette aus 2 Grundgliedern in Π-Schaltung





2. $f_1 = 2$kHz, Durchlaßbereich

$$a_B = \ln\sqrt{1 + \frac{1}{4}\left(\frac{Z_W}{Z_a} - \frac{Z_a}{Z_W}\right)^2 \sin^2 b_W}$$

$$b_B = \arctan\left[\frac{1}{2}\left(\frac{Z_W}{Z_a} + \frac{Z_a}{Z_W}\right)\tan b_W\right]$$

Zwischengrößen:

$$\Omega_{\mathrm{BS}} = \frac{-1}{\sqrt{\frac{L_r}{L_p}}\left(\frac{f}{f_0} - \frac{f_0}{f}\right)} = 0,28571$$

$$Z_W = \frac{X_D}{\sqrt{1 - \Omega_{\mathrm{BS}}^2}} = 834,80\Omega$$

$$b_W = 2 \cdot 2\arcsin\Omega = 66,40^\circ \quad \text{2 Grundglieder, jedes aus 2 Halbgliedern}$$

$$a_B = 1,3650 \cdot 10^{-2}\mathrm{Np}$$

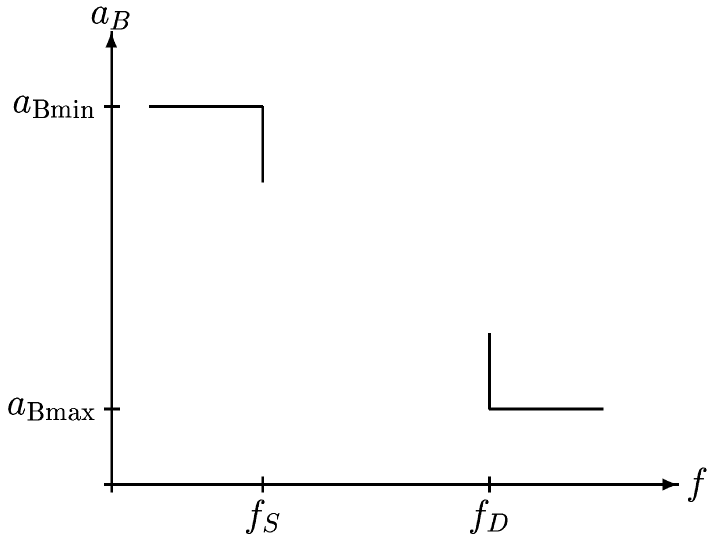
$$b_B = 66,74^\circ$$

$f_2 = 6$kHz, Sperrbereich

$$\begin{aligned} \Omega_{\mathrm{BS}} &= -6 \\ Z_W &= j135,22\Omega \\ a_W &= 9,9116\mathrm{Np} \\ a_B &= 10,544\mathrm{Np} \\ b_B &= -74,60^\circ \end{aligned}$$

Aufgabe 40:

Toleranzschema



Dimensionierung für ein Grundglied

$$\bar{a}(0) = a_{\text{Bmax}} = 0,3\text{Np}$$

$$Q = \mathrm{e}^{a_{\text{Bmax}}} + \sqrt{\mathrm{e}^{2a_{\text{Bmax}}} - 1} = 2,2565$$

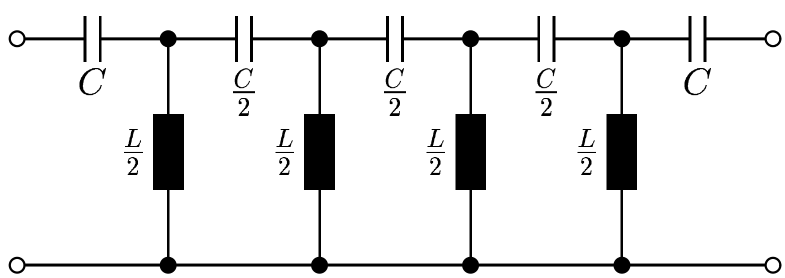
$$\Omega_D = -\sqrt{1 - \frac{1}{Q^4}} = -0,9805 \qquad \text{neg. Vorzeichen, da Hochpass}$$

$$f_g = -\Omega_D f_D = 3,432\text{kHz}$$

$$\Omega_S = -\frac{f_g}{f_S} = -1,716$$

$$n \geq \frac{arcosh\left(e^{a_{\text{Bmin}}}\right)}{2arcosh|\Omega_s|} = 3,8$$

$$\Rightarrow n = 4 \Rightarrow \text{ 4-gliedrige Kette}$$



Berechnung der Bauelemente

$$Q = \frac{X_D}{Z} \;\Rightarrow\; X_D = QZ = 2,2565\text{k}\Omega$$

Obige Formel gilt nur für T-Schaltung

$$X_D = 2\pi f_g L \;\Rightarrow\; L = \frac{X_D}{2\pi f_g} = 104,65\text{mH}$$

$$X_D = \sqrt{\frac{L}{C}} \;\Rightarrow\; C = \frac{L}{X_D^2} = 20,552\text{nF}$$

Aufgabe 41:

Warnung: Aufgabe ist überbestimmt :

$$f_0 = \sqrt{f_{D-} f_{D+}}$$
$$f_0 = \sqrt{f_{S-} f_{S+}}$$

Letzteres Gleichheitszeichen gilt nicht mehr, da alle 4 Frequenzen gegeben sind !

Dimensionierung für ein Grundglied

$$\bar{a}(0) = a_{\mathrm{Bmax}} = 0,3\mathrm{Np}$$
$$Q = \mathrm{e}^{\bar{a}} + \sqrt{\mathrm{e}^{2\bar{a}} - 1} = 2,2565$$
$$\Omega_D = \pm\sqrt{1 - \frac{1}{Q^4}} = \pm 0,9805$$
$$\Omega_{D-} = +0,9805 \quad \text{TP-Flanke}$$
$$\Omega_{D+} = -0,9805 \quad \text{HP-Flanke}$$
$$f_0 = \sqrt{f_{g+} f_{g-}} \quad \text{geht nicht, da } f_{g-},\ f_{g+} \text{ unbekannt}$$
$$f_0 = \sqrt{f_{D+} f_{D-}} = 5,6569\mathrm{kHz}$$
$$f = \frac{f_0}{2}\left(\frac{-1}{\Omega}\sqrt{\frac{L_p}{L_r}} + \sqrt{\frac{1}{\Omega^2}\frac{L_p}{L_r} + 4}\right) \quad \text{BS-Frequenznormierung}$$
$$f_{D+} = \frac{f_0}{2}\left(\frac{-1}{\Omega_{D+}}\sqrt{\frac{L_p}{L_r}} + \sqrt{\frac{1}{\Omega_{D+}^2}\frac{L_p}{L_r} + 4}\right)$$
$$\Delta f_D = f_{D+} - f_{D-} = \frac{f_0}{2}\left[\sqrt{\frac{L_p}{L_r}}\left(\frac{-1}{\Omega_{D+}} - \frac{-1}{\Omega_{D-}}\right)\right]$$
$$\sqrt{\frac{L_p}{L_r}} = \frac{2\Delta f_D}{f_0}\frac{\Omega_{D-}\Omega_{D+}}{\Omega_{D+} - \Omega_{D-}} = 2,4266$$

Grenzfrequenzen:

$$f_{g\pm} = \frac{f_0}{2}\left(\frac{-1}{\Omega_{g\pm}}\sqrt{\frac{L_p}{L_r}} + \sqrt{\frac{1}{\Omega_{g\pm}^2}\frac{L_p}{L_r} + 4}\right), \quad \Omega_{g\pm} = \pm 1$$
$$f_{g+} = 15,758\mathrm{kHz} \quad f_{g-} = 2,0308\mathrm{kHz}$$

$$\Delta f_g = f_{g+} - f_{g-} = 13,7272\text{kHz}$$

$$\Omega_{S\pm} = \frac{-1}{\sqrt{\frac{L_p}{L_r}}\left(\frac{f_{s\pm}}{f_0} - \frac{f_0}{f_{S\pm}}\right)}$$

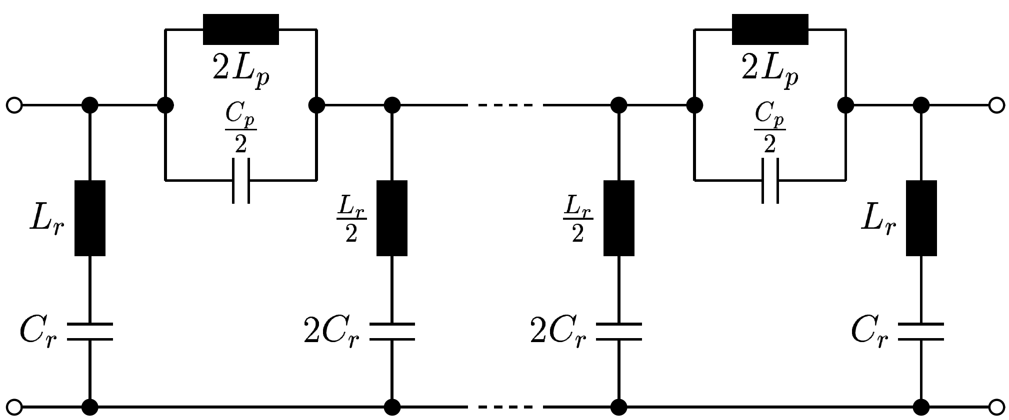
$$\Omega_{S+} = -1,0669 \qquad \Omega_{S-} = 1,3327$$

Zur Bestimmung der Grenzfrequenzen wurden die Durchlaßfrequenzen $f_{D\pm}$ symmetrisch zu f_0 gelegt. (Willkürliche Annahme, um weiterrechnen zu können). Infolgedessen können sich die Sperrfrequenzen $f_{S\pm}$ als unsymmetrisch herausstellen. (Durch die Angabe aller 4 Frequenzen ist die Aufgabe überbestimmt!)

Die weiteren Berechnungen müssen mit der kritischsten Frequenz erfolgen, also mit derjenigen, deren Betrag $|\Omega_S|$ am nächsten an 1 liegt. Bei der weniger kritischen Sperrfrequenz ergibt sich somit eine Reserve.

$$n \geq \frac{arcosh\left(e^{a_{\text{Bmin}}}\right)}{2arcosh|\Omega_s|} = 6,5$$

$$\Rightarrow n = 7 \Rightarrow \text{ 7-gliedrige Kette}$$



Berechnung der Bauelemente

$$Q = \frac{Z}{X_D} \;\Rightarrow\; X_D = \frac{Z}{Q} = 443,16\Omega$$

Obige Formel gilt nur für π-Schaltung

$$X_D = 2\pi\Delta f_g L_r \;\Rightarrow\; L_r = \frac{X_D}{2\pi\Delta f_g} = 5,1381\text{mH}$$

$$X_D = \sqrt{\frac{L_r}{C_p}} \;\Rightarrow\; C_p = \frac{L_r}{X_D^2} = 26,162\text{nF}$$

$$\omega_0 = \frac{1}{\sqrt{L_p C_p}} \;\Rightarrow\; L_p = \frac{1}{4\pi^2 f_0^2 C_p} = 30,256\text{mH}$$

$$\sqrt{\frac{C_r}{C_p}} = \sqrt{\frac{L_p}{L_r}} \;\Rightarrow\; C_r = \frac{C_p L_p}{L_r} = 154,06\text{nF}$$

Aufgabe 42:

Zerlegen der komplexen Beziehung in zwei reelle Beziehungen

$$\begin{aligned} \text{Betrag:} \quad & \underline{\hat{u}} = \underline{p}L\underline{\hat{i}} \\ \text{Phase:} \quad & \varphi_u = \varphi_i + \psi \\ \text{mit:} \quad & \underline{p} = p\mathrm{e}^{j\psi} \\ & p = \sqrt{\sigma^2 + \omega^2} \\ & \psi = \arctan\frac{\omega}{\sigma} \qquad \text{für } \sigma > 0 \end{aligned}$$

Warnung: Der Zusammenhang gilt zwar allgemein, die Herleitung hier gilt aber nur für $\sigma > 0$!
Gleichung für die Induktivität:

$$u(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

$$\begin{aligned} i(t) &= \hat{i}\mathrm{e}^{\sigma t}\cos(\omega t + \varphi_i) \\ u(t) &= L\hat{i}\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{\sigma t}\cos(\omega t + \varphi_i)\right) \\ &= L\hat{i}\mathrm{e}^{\sigma t}\left(\sigma\cos(\omega t + \varphi_i) - \omega\sin(\omega t + \varphi_i)\right) \\ & \quad \text{mit } a\sin\omega t + b\cos\omega t = \sqrt{a^2 + b^2}\sin(\omega t + \arctan\frac{b}{a}) \quad \text{(s. Bronstein)} \\ u(t) &= L\hat{i}\mathrm{e}^{\sigma t}\left[\sqrt{\sigma^2 + \omega^2}\cos\left(\omega t + \varphi_i + \arctan\frac{\omega}{\sigma}\right)\right] \qquad \text{für } \sigma > 0 \\ &= L\hat{i}\mathrm{e}^{\sigma t}\left[p\cos(\omega t + \varphi_i + \psi)\right] \\ &= pL\hat{i}\mathrm{e}^{\sigma t}\cos(\omega t + \varphi_i + \psi) \\ &= \hat{u}\mathrm{e}^{\sigma t}\cos(\omega t + \varphi_u) \\ &\Rightarrow \hat{u} = pL\hat{i} \quad ; \qquad \varphi_u = \varphi_i + \psi \end{aligned}$$

Der Betrag der Spannung $\hat{u}$ ist proportional zu L, $\sqrt{\sigma^2 + \omega^2}$ und $\hat{i}$. Die Phase ergibt sich für $\sigma > 0$ aus $\varphi_i + \arctan\frac{\omega}{\sigma}$.

Aufgabe 43:

1. Übertragungsfuktion

$$A(p) = \frac{U_2}{U_1} = \frac{U_2}{U_{C1}} \cdot \frac{U_{C1}}{U_1}$$

$$\frac{U_2}{U_{C1}} = \frac{\frac{1}{pC_2}}{R_2 + \frac{1}{pC_2}} = \frac{1}{1 + pR_2C_2}$$

$$\frac{U_{C1}}{U_1} = \frac{Z}{R_1 + Z}$$

$$Z = \frac{1}{pC_1 + \frac{1}{R_2 + \frac{1}{pC_2}}} = \frac{1}{pC_1 + \frac{pC_2}{1 + pR_2C_2}} = \frac{1 + pR_2C_2}{pC_1 + pC_2 + p^2R_2C_1C_2}$$

$$\frac{U_{C1}}{U_1} = \frac{\frac{1 + pR_2C_2}{pC_1 + pC_2 + p^2R_2C_1C_2}}{R_1 + \frac{1 + pR_2C_2}{pC_1 + pC_2 + p^2R_2C_1C_2}}$$

$$= \frac{1 + pR_2C_2}{pR_1C_1 + pR_1C_2 + p^2R_1R_2C_1C_2 + 1 + pR_2C_2}$$

$$= \frac{1 + pR_2C_2}{1 + p(R_1C_1 + R_1C_2 + R_2C_2) + p^2R_1R_2C_1C_2}$$

$$A(p) = \frac{U_2}{U_{C1}} \cdot \frac{U_{C1}}{U_1}$$

$$= \frac{1}{1 + pR_2C_2} \cdot \frac{1 + pR_2C_2}{1 + p(R_1C_1 + R_1C_2 + R_2C_2) + p^2R_1R_2C_1C_2}$$

$$= \frac{1}{1 + p(R_1C_1 + R_1C_2 + R_2C_2) + p^2R_1R_2C_1C_2}$$

2. Normierung

$$P = \frac{p}{\omega_0} \qquad p = P\omega_0$$

$$A(P) = \frac{1}{1 + P\omega_0(R_1C_1 + R_1C_2 + R_2C_2) + P^2\omega_0^2R_1R_2C_1C_2}$$

$$= \frac{1}{1 + a_1 P + b_1 P^2}$$

$$a_1 = \omega_0 (R_1 C_1 + R_1 C_2 + R_2 C_2)$$

$$b_1 = \omega_0^2 R_1 R_2 C_1 C_2$$

3. Pole

$$A(P) = \frac{\frac{1}{b_1}}{\frac{1}{b_1} + \frac{a_1}{b_1} P + P^2} = \frac{\frac{1}{b_1}}{(P - P_{\infty 1})(P - P_{\infty 2})}$$

$P_{\infty 1,2}$ sind die Nullstellen von:

$$P^2 + \frac{a_1}{b_1} P + \frac{1}{b_1} = 0$$

$$P_{\infty 1,2} = \frac{a_1}{2b_1} \left(-1 \pm \sqrt{1 - \frac{4b_1}{a_1^2}} \right)$$

4. Pol-Nullstellen-Diagramm
Festlegung der Normierung so, daß $\frac{a_1}{b_1} = 1$ wird. (Willkürliche Festlegung)

$$\frac{a_1}{b_1} = 1 \Rightarrow \frac{R_1 C_1 + R_1 C_2 + R_2 C_2}{\omega_0 R_1 R_2 C_1 C_2} = 1$$

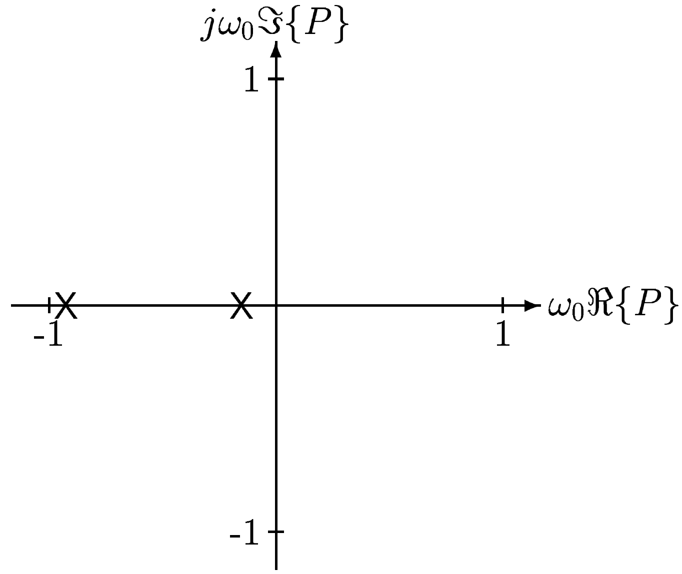
$$\Rightarrow \omega_0 = \frac{R_1 (C_1 + C_2) + R_2 C_2}{R_1 R_2 C_1 C_2} = 8500 s^{-1}$$

$$a_1 = \omega_0 (R_1 C_1 + R_1 C_2 + R_2 C_2) = 14,45$$

$$P_{\infty 1,2} = \frac{1}{2} \left(-1 \pm \sqrt{1 - \frac{4}{a_1}} \right) = \frac{1}{2} (-1 \pm 0,8504)$$

$$P_{\infty 1} = -0,9252 \qquad P_{\infty 2} = -0,0748$$

Pol-Nullstellen-Diagramm


$j\omega_0\Im\{P\}$
1
-1
$\omega_0\Re\{P\}$
-1
1

Aufgabe 44:

1. Grenzfrequenz

$$\left|\frac{A(p=j\omega_g)}{A(p=0)}\right| = \frac{1}{\sqrt{2}} \quad \text{oder} \quad \left|\frac{A(p=j\omega_g)}{A(p=0)}\right|^2 = \frac{1}{2}$$

$$A(p) = \frac{1}{1+p(R_1C_1+R_1C_2+R_2C_2)+p^2R_1R_2C_1C_2} \quad \text{siehe Aufg. 43}$$

Zahlenwerte:

$$R_1C_1+R_1C_2+R_2C_2 = 1,7\cdot 10^{-3}s$$

$$R_1R_2C_1C_2 = 0,2\cdot 10^{-6}s^2$$

$$= \frac{1}{1+j\omega\cdot 1,7\cdot 10^{-3}s-\omega^2\cdot 0,2\cdot 10^{-6}s^2}$$

$$A(0) = 1$$

$$\frac{1}{2} = \left|1+\frac{1}{+j\omega_g\cdot 1,7\cdot 10^{-3}s-\omega_g^2\cdot 0,2\cdot 10^{-6}s^2}\right|^2$$

$$2 = \left|1+j\omega_g\cdot 1,7\cdot 10^{-3}s-\omega_g^2\cdot 0,2\cdot 10^{-6}s^2\right|^2$$

$$2 = \left(1-\omega_g^2\cdot 0,2\cdot 10^{-6}s^2\right)^2+\left(\omega_g\cdot 1,7\cdot 10^{-3}s\right)^2$$

$$2 = 1-\omega_g^2\cdot 0,4\cdot 10^{-6}s^2+\omega_g^4\cdot 0,04\cdot 10^{-12}s^4+\omega_g^2\cdot 2,89\cdot 10^{-6}s^2$$

$$2 = 1+2,49\cdot 10^{-6}s^2\omega_g^2+0,04\cdot 10^{-12}s^4\omega_g^4$$

$$0 = \omega_g^4+62,25\cdot 10^6s^{-2}\omega_g^2-25\cdot 10^{12}s^{-4}$$

$$\omega_{g1,2}^2 = -31,125\cdot 10^6s^{-2}\pm\sqrt{968,77\cdot 10^{12}s^{-4}+25\cdot 10^{12}s^{-4}}$$

$$\omega_{g1,2}^2 = -31,125\cdot 10^6s^{-2}\pm 31,.524\cdot 10^6s^{-2}$$

$$\omega_{g,1}^2 = 0,399\cdot 10^6s^{-2}$$

$$\omega_g = 631,7s^{-1} \quad f_g = 100,5\text{Hz}$$

2. Normierung auf $\omega_0 = \omega_g$

$$P = \frac{p}{\omega_g}$$

$$A(P) = \frac{1}{1+a_1P+b_1P^2}$$

mit: $a_1 = \omega_g(R_1C_1 + R_1C_2 + R_2C_2) = 1,0739$

$b_1 = \omega_g^2 R_1 R_2 C_1 C_2 = 79,81 \cdot 10^{-3}$

$$A(P) = \frac{1}{1 + 1,0739P + 0,07981P^2}$$

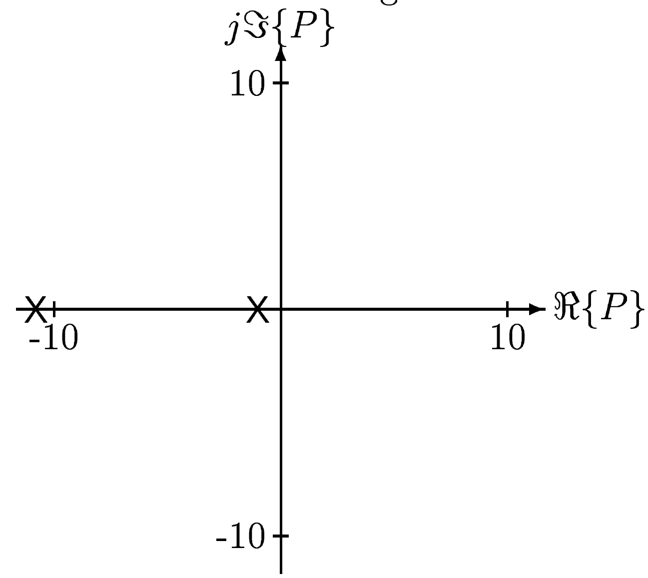
3. Polstellen

$$\begin{aligned} P^2 + \frac{a_1}{b_1}P + \frac{1}{b_1} &= 0 \\ P^2 + 13,456P + 12,53 &= 0 \\ P_{\infty 1,2} &= -6,728 \pm \sqrt{45,266 - 12,53} \\ &= -6,728 \pm 5,722 \\ P_{\infty 1} &= -1,006 \qquad P_{\infty 2} = -12,45 \end{aligned}$$

4. Linearfaktoren

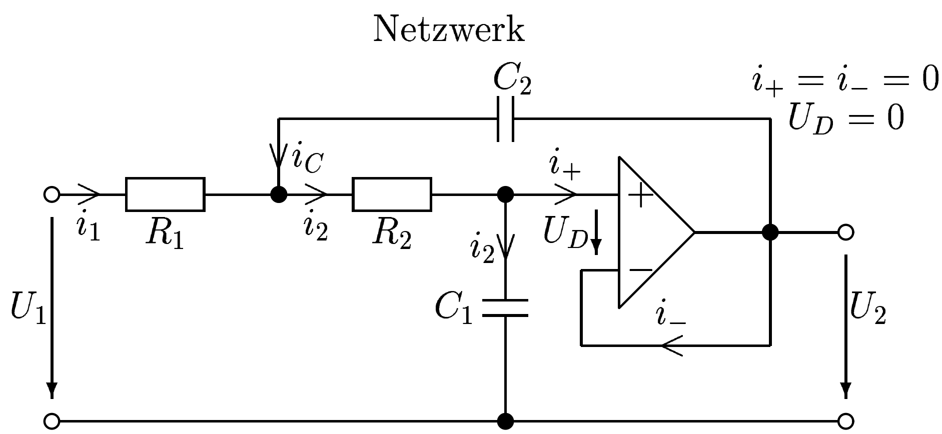
$$A(P) = \frac{12,530}{(P + 1,006)(P + 12,45)}$$

Pol-Nullstellen-Diagramm



Aufgabe 45:

1. Übertragungsfunktion
   $A(P) = U_2/U_1$ aus einer Maschenanalyse des Netzwerks.
   Der Operationsverstärker sei ideal (kein Eingangsstrom, keine Potentialdifferenz zwischen den Eingängen).



$$\text{K:}\qquad i_1 + i_C - i_2 = 0$$

$$\text{M1:}\qquad i_1 R_1 - \frac{i_C}{pC_2} + u_2 - u_1 = 0$$

$$\text{M2:}\qquad \frac{i_C}{pC_2} + i_2 R_2 = 0$$

$$\text{M3:}\qquad u_2 - \frac{i_2}{pC_1} = 0$$

$$i_2 = u_2 p C_1 \quad \text{aus M3}$$

$$i_C = -pC_2R_2i_2 \quad \Rightarrow i_C = -u_2p^2R_2C_1C_2 \qquad \text{aus M2}$$

$$i_1 = i_2 - i_C \quad \Rightarrow i_1 = u_2(1 + pR_2C_1)pC_2 \qquad \text{aus K}$$

$$u_2(1 + pR_2C_2)pC_1 \cdot R_1 + u_2pR_2C_1 + u_2 = u_1 \qquad \text{aus M1}$$

$$\frac{u_2}{u_1} = [(1 + pR_2C_2)pC_1R_1 + pR_2C_1 + 1] = 1$$

$$\frac{u_2}{u_1} = \frac{1}{(1 + pR_2C_2)pC_1R_1 + pR_2C_1 + 1}$$

$$\frac{u_2}{u_1} = \frac{1}{1 + p(R_1 + R_2)C_1 + p^2R_1R_2C_1C_2} = A(p)$$

2. Normierung

$$A(p) = \frac{1}{1 + \frac{p}{\omega_0}\omega_0(R_1+R_2)C_1 + \frac{p^2}{\omega_0^2}\omega_0^2 R_1R_2C_1C_2}$$

$$A(P) = \frac{1}{1 + a_1 P + b_1 P^2}$$

mit:

$$a_1 = \omega_0(R_1+R_2)C_1$$

$$b_1 = \omega_0^2 R_1R_2C_1C_2$$

$$P = \frac{p}{\omega_0}$$

3. Grenzfrequenz

$$A(j\Omega) = \frac{1}{1 + a_1 j\Omega - b_1\Omega^2}$$

$$|A(j\Omega)|^2 = \frac{1}{(1-b_1\Omega^2)^2 + a_1^2\Omega^2}$$

$$A(0) = 1$$

Normierung mit $\omega_0 = \omega_g$

$$|(A(j1)|^2 = \frac{1}{(1-b_1)^2 + a_1^2} = \frac{1}{1 - 2b_1 + b_1^2 + a_1^2}$$

$$\left|\frac{A(j1)}{A(0)}\right|^2 = \frac{1}{2}$$

$$2 = 1 - 2b_1 + b_1^2 + a_1^2$$

$$a_1 = \omega_g 35 \cdot 10^{-5} s$$

$$b_1 = \omega_g^2 \cdot 10^{-7} s^2$$

$$0 = \omega_g^4 \cdot 10^{-14} s^4 - 0,775 \cdot 10^{-7} s^2 \omega_g^2 - 1$$

$$0 = \omega_g^4 - 0,775 \cdot 10^7 s^{-2} \omega_g^2 - 10^{14} s^{-4}$$

$$\omega_g^2 = 3,875 \cdot 10^7 s^{-2} \pm \sqrt{15,016 \cdot 10^{12} s^4 + 100 \cdot 10^{12} s^4}$$

$$= 3,875 \cdot 10^6 s^{-2} \pm 10,725 \cdot 10^6 s^{-2}$$

$$\omega_g^2 = 14,6 \cdot 10^6 s^{-2}$$

$$\omega_g = 3,821 \cdot 10^3 s^{-1}$$

4. Zerlegung in Linearfaktoren

$$a_1 = \omega_g 35 \cdot 10^{-5} s = 1,3373$$

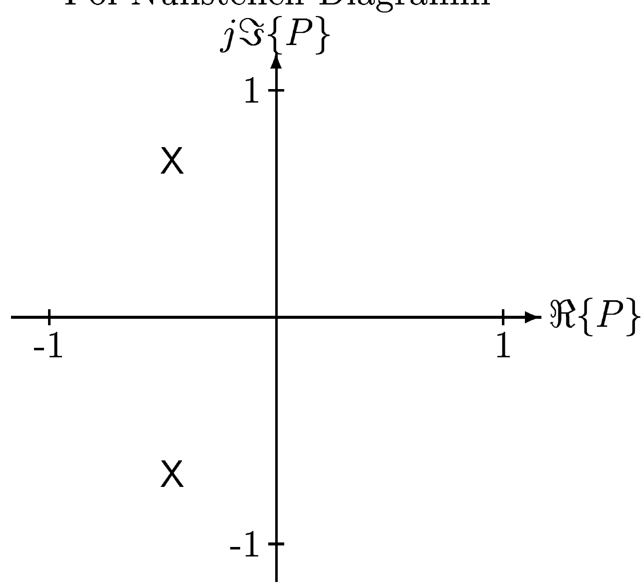
$$b_1 = \omega_g^2 \cdot 10^{-7} s^2 = 1,4600$$

$$A(P) = \frac{1}{1 + 1,3373P + 1,4600P^2} = \frac{0,6849}{0,6849 + 0,9169P + P^2}$$

$$P_{\infty 1,2} = -0,4580 \pm \sqrt{0,2098 - 0,6849} = -0,4580 \pm j0,6893$$

$$A(P) = \frac{0,6849}{(P + 0,4580 - j0,6893)(P + 0,4580 + j0,6893)}$$

Pol-Nullstellen-Diagramm



Aufgabe 46:

1. Übertragungsfunktion $A(P)$

$$A(P) = \frac{A_0}{1 + c_1 P + c_2 P^2 + c_3 P^3} \qquad \text{TP 3.Ordnung}$$

$$A(j\Omega) = \frac{A_0}{1 + c_1 j\Omega + c_2 (j\Omega)^2 + c_3 (j\Omega)^3} \qquad \text{Frequenzgang}$$

$$= \frac{A_0}{1 - c_2\Omega^2 + j\left(c_1\Omega - c_3\Omega^3\right)}$$

$$|A(j\Omega)|^2 = \frac{A_0^2}{1 + \left(c_1^2 - 2c_2\right)\Omega^2 + \left(c_2^2 - 2c_1c_3\right)\Omega^4 + c_3^2\Omega^6}$$

Beim Butterworth-Tiefpaß soll der Verlauf von $|A(j\Omega)|^2$ möglichst lange konstant sein. Dies wird erreicht, wenn die Koeffizienten von Ω^2 und Ω^4 Null sind. Der Koeffizient von Ω^6 ergibt sich aus der Bedingung für die Grenzfrequenz.

$$c_1^2 - 2c_2 = 0$$
$$c_2^2 - 2c_1c_3 = 0$$
$$c_3^2 = 1$$

Diese Gleichungen führen auf vier verschiedene Lösungen

$$\vec{c_1} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \qquad \vec{c_2} = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}, \qquad \vec{c_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ oder } \quad \vec{c_4} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

die jedoch alle auf den gleichen Betragsverlauf hinauslaufen:

$$|A(P)| = \frac{A_0^2}{\Omega^6}$$

Also

$$A(P) = \frac{A_0}{1 + 2P + 2P^2 + P^3} \quad \vee \quad A(P) = \frac{A_0}{1 + P^3}$$

$$|A(j\Omega)|^2 = \frac{A_0^2}{1 + \Omega^6}$$

2. Zerlegung

$$A(P) = \frac{A_0}{(1 + a_1 P + b_1 P^2)\,(1 + a_2 P + b_2 P^2)} \quad b_2 = 0, \text{ da kein } P^4$$
$$= \frac{A_0}{\prod\limits_{i=1}^{3} (P - P_{\infty i})}$$

Nullstellen von $1 + 2P + 2P^2 + P^3$:

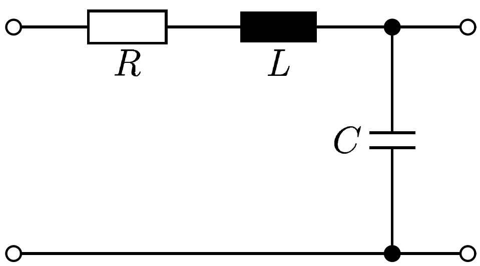
$$P_{\infty 1} = -1 \qquad \text{raten}$$
$$(P^3 + 2P^2 + 2P + 1) : (P + 1) = P^2 + P + 1 \quad \text{Polynomdivision}$$
$$A(P) = \frac{A_0}{(1 + P + P^2)(1 + P)}$$
$$a_1 = 1 \quad b_1 = 1 \quad a_2 = 1 \quad b_2 = 0$$

Aufgabe 47:

1. Bessel-Tiefpaß, Übertragungsfunktion laut Tabelle (z.B. Tietze, Schenk (8. Auflage, Kap. 14.1.4, Seite 391 bis 397)

$$A(P) = \frac{A_0}{1 + 1,3617P + 0,6180P^2}$$

Diese Übertragungsfunktion läßt sich durch einen RLC-Tiefpaß realisieren (siehe Vorlesung).



$$A(P) = \frac{1}{1 + \omega_g RCP + \omega_g^2 LCP^2}$$

Koeffizientenvergleich (C=10μF wird frei gewählt).

$$A_0 = 1$$

$$\omega_g RC = 1,3617 \;\Rightarrow\; R = \frac{1,3617}{2\pi f_g C} = 108,36\Omega$$

$$\omega_g^2 LC = 0,6180 \;\Rightarrow\; L = \frac{0,6180}{4\pi^2 f_g^2 C} = 39,135\text{mH}$$

2. Butterworth-Tiefpaß, Übertragungsfunktion laut Tabelle

$$A(P) = \frac{A_0}{1 + 1,4142P + P^2}$$

Koeffizientenvergleich (C=10μF wird frei gewählt).

$$A_0 = 1$$

$$\omega_g RC = 1,4142 \;\Rightarrow\; R = \frac{1,4142}{2\pi f_g C} = 112,54\Omega$$

$$\omega_g^2 LC = 1 \;\Rightarrow\; L = \frac{1}{4\pi^2 f_g^2 C} = 63,326\text{mH}$$

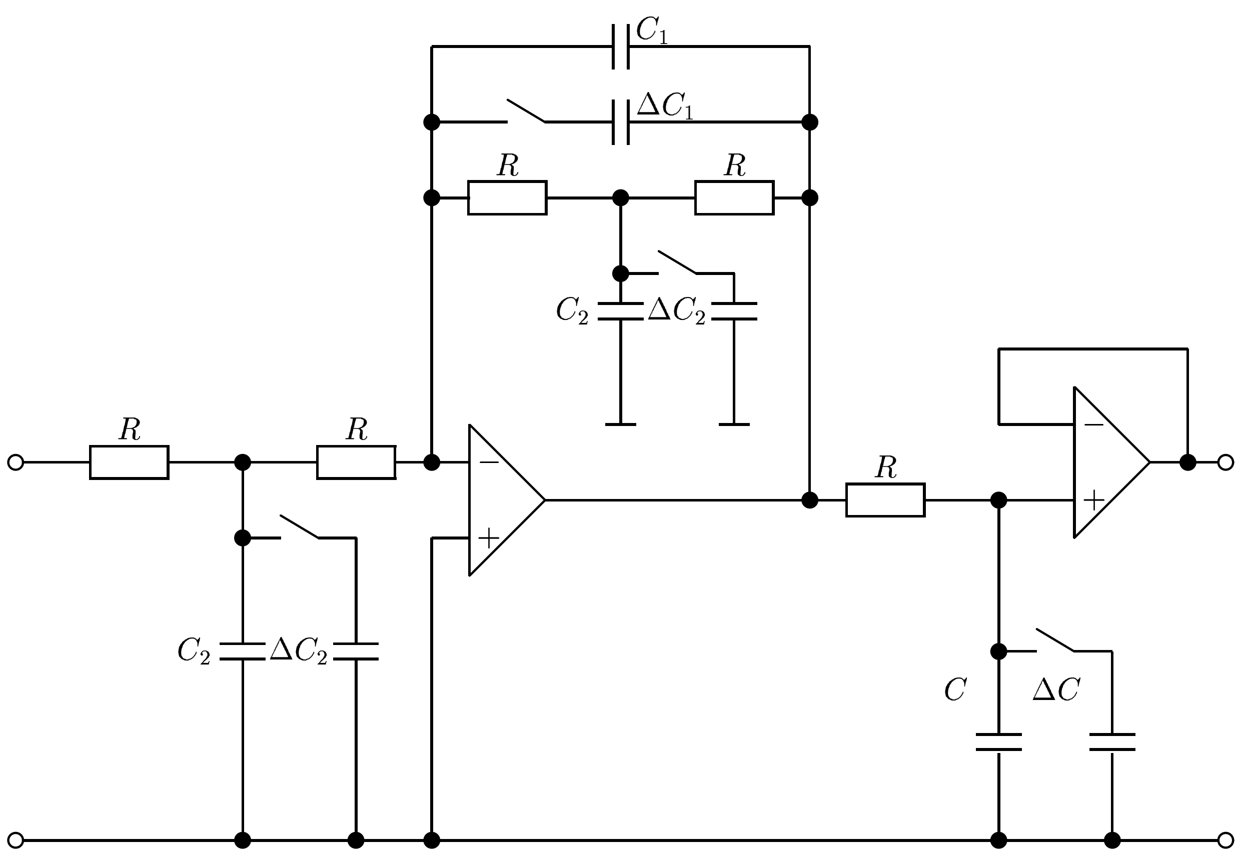
Aufgabe 48:

Aufbau: Kettenschaltung zweier (entkoppelter) Blöcke 2. und 1. Ordnung. 1. Block: einfach gegengekoppelter OP-Verstärker, 2. Block: RC-Glied

Übertragungsfunktion (laut Tabelle): $a_1 = 0,3559$, $b_1 = 1,1923$, $a_2 = 3,3496$, $b_2 = 0$

$$A(P) = \frac{A_0}{(1 + 0,3559P + 1,1923P^2)(1 + 3,3496P)}$$

Schaltung:



Die Umschaltung der Grenzfrequenz erfolgt durch eine Zuschaltung der Kapazitäten ΔC, ΔC_1, ΔC_2 statt durch Umschaltung.

$$A_1(P) = \frac{-1}{1 + 2\omega_g RC_1 P + (\omega_g^2 R^2 C_1 C_2) P^2}$$

$$A_2(P) = \frac{1}{1 + \omega_g RCP}$$

Auslegung der Kapazitäten C, C_1 und C_2 für die höhere Grenzfrequenz $f_g = f_{g2} = 5000\text{Hz}$, da diese durch Zuschalten von Kapazitäten nur erniedrigt werden kann.

$$a_1 = 2 \cdot 2\pi f_g R C_1 \;\Rightarrow\; C_1 = \frac{a_1}{4\pi f_g R} = 566,43\text{pF}$$

$$b_1 = 4\pi^2 f_g^2 R^2 C_1 C_2 \;\Rightarrow\; C_2 = \frac{b_1}{4\pi^2 f_g^2 R^2 C_1} = 21,327\text{nF}$$

$$a_2 = 2\pi f_g R C \;\Rightarrow\; C = \frac{a_2}{2\pi f_g R} = 10,662\text{nF}$$

$f_g = f_{g1} = 3400\text{Hz}$

$$C_1' = 833\text{pF} \;\rightarrow\; \Delta C_1 = 266,56\text{pF}$$

$$C_2' = 46,122\text{nF} \;\rightarrow\; \Delta C_2 = 24,795\text{nF}$$

$$C' = 15,68\text{nF} \;\rightarrow\; \Delta C = 5,018\text{nF}$$

Aufgabe 49:

Siehe auch Aufg.45!

1. Vergleich mit den Standardschaltungen: Tiefpaß 4. Ordnung
2. einfach mitgekoppelt, $\alpha = 1$
3. Berechnung a_ν, b_ν
   Übertragungsfunktion für den TP 4. Ordnung

$$A(P) = \frac{A_0}{(1 + a_1 P + b_1 P^2)(1 + a_2 P + b_2 P^2)}$$

$$\begin{aligned} a_1 &= \omega_g(R_1C_1 + R_2C_2) \\ a_2 &= \omega_g(R_1'C_1' + R_2'C_2') \\ b_1 &= \omega_g^2 R_1R_2C_1C_2 \\ b_2 &= \omega_g^2 R_1'R_2'C_1'C_2' \end{aligned}$$

mit $R_1 = R_2 = R_1' = R_2' = 10\text{k}\Omega$, $C_1 = 528,73\text{pF}$, $C_2 = 44,64\text{nF}$, $C_1' = 5,351\text{nF}$, $C_2' = 18,49\text{nF}$ und unbekanntem ω_g. Einsetzen der Bauelemente ergibt:

$$\begin{aligned} a_1 &= 2\omega_g R_1C_1 = \omega_g \cdot 10,575 \cdot 10^{-6}s \\ a_2 &= 2\omega_g R_1'C_1' = \omega_g \cdot 107,02 \cdot 10^{-6}s \\ b_1 &= \omega_g^2 R_1^2 C_1C_2 = \omega_g^2 \cdot 2,3603 \cdot 10^{-9}s^2 \\ b_2 &= \omega_g^2 R_1^2 C_1'C_2' = \omega_g^2 \cdot 9,8940 \cdot 10^{-9}s^2 \end{aligned}$$

Da ω_g unbekannt ist, können die Koeffizienten nicht direkt berechnet und mit denen der Tabelle verglichen werden, um den Filtertyp zu ermitteln. Die Verhältnisse a_1/a_2 und b_1/b_2 sind jedoch unabhängig von der Grenzfrequenz ω_g und können somit verwendet werden.

$$\begin{aligned} \frac{a_1}{a_2} &= 0,098813 \\ \frac{b_1}{b_2} &= 0,23856 \end{aligned}$$

Beim Vergleich mit den Koeffizientenverhältnissen aus den Tabellen ist zu beachten, daß die Blöcke 1 und 2 hier vertauscht sind! Also

$$\frac{a_1}{a_2}_{\text{Tabelle}} = 10,1201$$

$$\frac{b_1}{b_2}_{\text{Tabelle}} = 4,1918$$

Der Vergleich ergibt, daß es sich um ein **Tschebyscheff**-Filter mit der **Welligkeit 2dB** handelt.

4. Grenzfrequenz
   Nachdem der Filtertyp bekannt ist, können die Koeffizienten a_ν, b_ν aus der Tabelle entnommen werden (Achtung: Blöcke hier im Vergleich zur Tabelle vertauscht!).

$$a_1 = 0,2374 \quad b_1 = 1,1896 \quad a_2 = 2,4025 \quad b_2 = 4,8962$$

   damit

$$\omega_g = \frac{a_1}{107,02 \cdot 10^{-6} s} = 2,218\text{kHz}$$

Aufgabe 50:

Tiefpaß-Hochpaß-Transformation

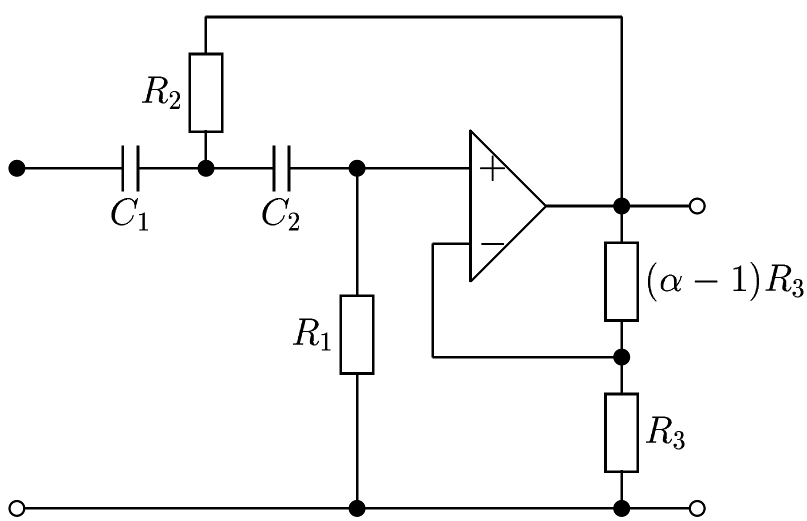
$$P_{\mathrm{HP}} = \frac{1}{P_{\mathrm{TP}}}$$

$$A(P) = \frac{A_\infty}{\prod_\nu \left(1 + \frac{a_\nu}{P} + \frac{b_\nu}{P^2}\right)}$$

Tabelle: $a_1 = 1,3617 \qquad b_1 = 0,6180$

$$A(P) = \frac{A_\infty}{1 + \frac{1,3617}{P} + \frac{0,6180}{P^2}}$$

Realisierung durch einfach mitgekoppelte OP-Schaltung. HP-Schaltungen können aus TP-Schaltungen durch Vertauschen von Widerständen und Kondensatoren in den frequenzbestimmenden Schaltungsteilen gewonnen werden.



$$A(P) = \frac{\alpha}{1 + \frac{R_2(C_1+C_2) + R_1C_2(1-\alpha)}{R_1R_2C_1C_2\omega_g} \cdot \frac{1}{P} + \frac{1}{R_1R_2C_1C_2\omega_g^2} \cdot \frac{1}{P^2}}$$

$$= \frac{1}{1 + \frac{R_2(C_1+C_2)}{R_1R_2C_1C_2\omega_g} \cdot \frac{1}{P} + \frac{1}{R_1R_2C_1C_2\omega_g^2} \cdot \frac{1}{P^2}} \qquad \alpha = 1$$

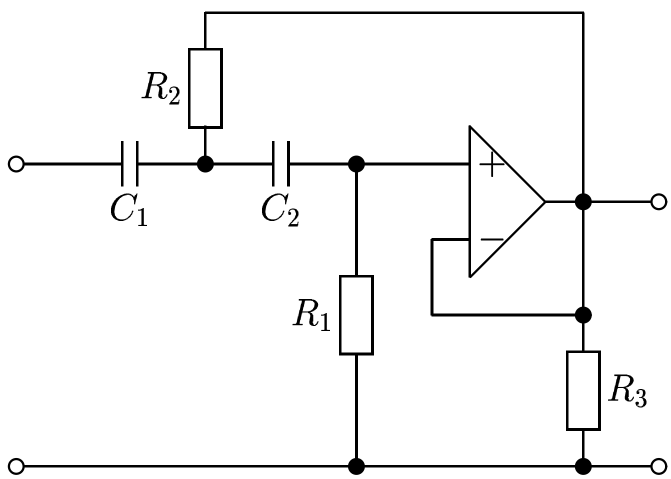
$$a_1 = \frac{R_2(C_1+C_2)}{R_1R_2C_1C_2\omega_g}$$

$$b_1 = \frac{1}{R_1 R_2 C_1 C_2 \omega_g^2}$$

zur Vereinfachung: $C_1 = C_2 = C = 100\text{nF}$

$$R_1 = \frac{1}{\pi f_g C a_1} = 2337,59\Omega$$

$$R_2 = \frac{1}{4\pi f_g C b_1} = 5150,65\Omega$$



Aufgabe 51:

Tiefpaß-Bandpaß-Transformation

$$P_{\mathrm{BP}} = \frac{1}{\Delta\Omega}\left(P_{\mathrm{TP}} + \frac{1}{P_{\mathrm{TP}}}\right)$$

Einsetzen der Transformation in den Tiefpaß 1. Ordnung

$$A(P) = \frac{A_0}{1 + a_1 P}$$

ergibt

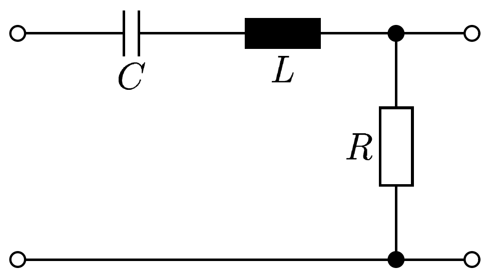
$$A(P) = \frac{A_0}{1 + a_1 \frac{1}{\Delta\Omega}\left(P + \frac{1}{P}\right)} = \frac{A_0 \Delta\Omega P}{a_1 + \Delta\Omega P + a_1 P^2}$$

Tabelle: $a_1 = 1,0000$ Verdopplung der Ordnung !

$$A(P) = \frac{A_0 \Delta\Omega P}{1 + \Delta\Omega P + P^2}$$

Realisierungen

1. LRC-Bandpaßfilter



$$A(P) = \frac{R\sqrt{\frac{C}{L}}P}{1 + R\sqrt{\frac{C}{L}}P + P^2} \quad ; \quad P = \frac{p}{\omega_0} \quad ; \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Delta\Omega = \frac{f_+ - f_-}{f_0} = R\sqrt{\frac{C}{L}}$$

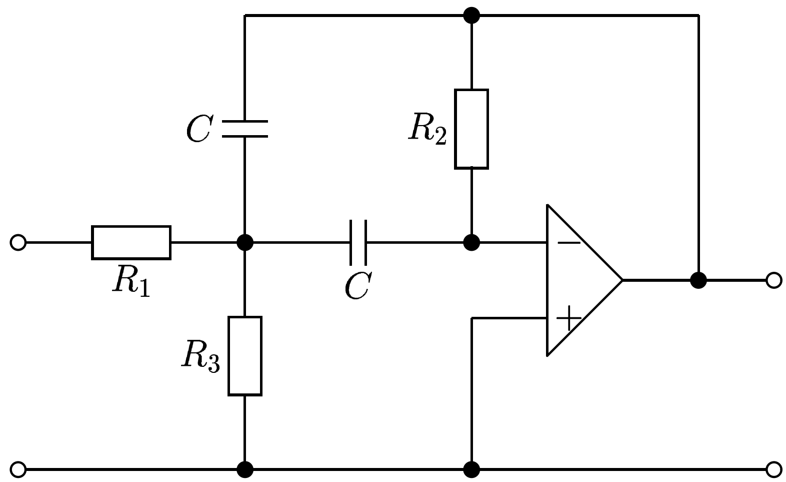
$C = 10\mathrm{nF}$, frei gewählt

$$f_0 = \sqrt{f_+ f_-} = 979,80\mathrm{Hz}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{4\pi^2 f_0^2 C} = 26,39\mathrm{mH}$$

$$R = \frac{f_+ - f_-}{f_0}\sqrt{\frac{L}{C}} = 198,94\Omega$$

2. mehrfach gegengekoppelte OP-Schaltung



$$A(P) = \frac{-\frac{R_2R_3}{R_1+R_2}C\omega_r P}{1 + 2\frac{2R_1R_3}{R_1+R_3}C\omega_r P + \frac{R_1R_2R_3}{R_1+R_3}C^2\omega_r^2P^2}$$

$$\omega_r = \frac{1}{C}\sqrt{\frac{R_1+R_2}{R_1R_2R_3}}$$

$$A(P) = \frac{-\sqrt{\frac{R_2R_3}{R_1(R_1+R_3)}}P}{1 + 2\sqrt{\frac{R_1R_3}{R_2(R_1+R_3}}P + P^2}$$

$$\sqrt{\frac{R_2R_3}{R_1(R_1+R_3)}} = 2\sqrt{\frac{R_1R_3}{R_2(R_1+R_3)}}$$

$$R_2^2R_3(R_1+R_3) = 4R_1^2R_3(R_1+R_3)$$

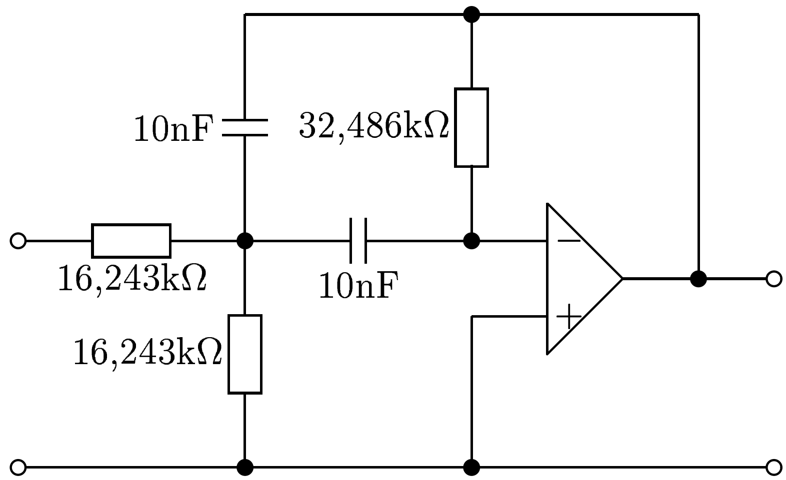
$$R_2^2 = 4R_1^2 \Rightarrow R_2 = 2R_1$$

$$R_3 = R_1 \quad \text{und} \quad C = 10\text{nF, frei gewählt}$$

$$\Delta\Omega = \sqrt{\frac{R_2R_3}{R_1(R_1+R_3)}} = \sqrt{\frac{2R_1^2}{R_1\cdot 2R_1}} = 1$$

$$f_r = \frac{1}{2\pi C}\sqrt{\frac{2R_1}{2R_1^3}} = \frac{1}{2\pi CR_1}$$

$$R_1 = \frac{1}{2\pi f_r C} = 16,243\text{k}\Omega$$



Aufgabe 52:

$$\delta(t) \overset{\mathcal{L}}{\circ\!\!-\!\!\bullet} 1 \qquad \varepsilon(t) \overset{\mathcal{L}}{\circ\!\!-\!\!\bullet} \frac{1}{p}$$

1. TP 1. Ordnung

$$A(p) = \frac{1}{p + \frac{1}{RC}}$$

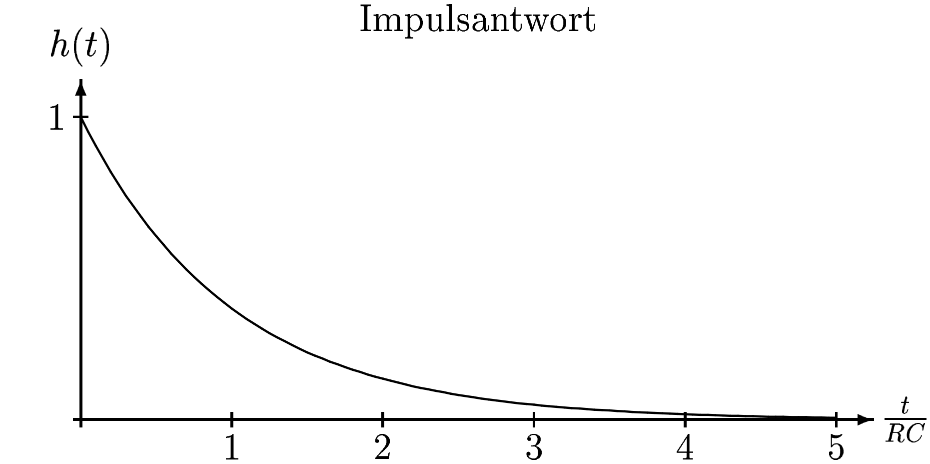
$$h(t) = \mathcal{L}^{-1}\left\{\frac{1}{p + \frac{1}{RC}}\right\}$$

$$= \mathrm{e}^{-\frac{t}{RC}}$$

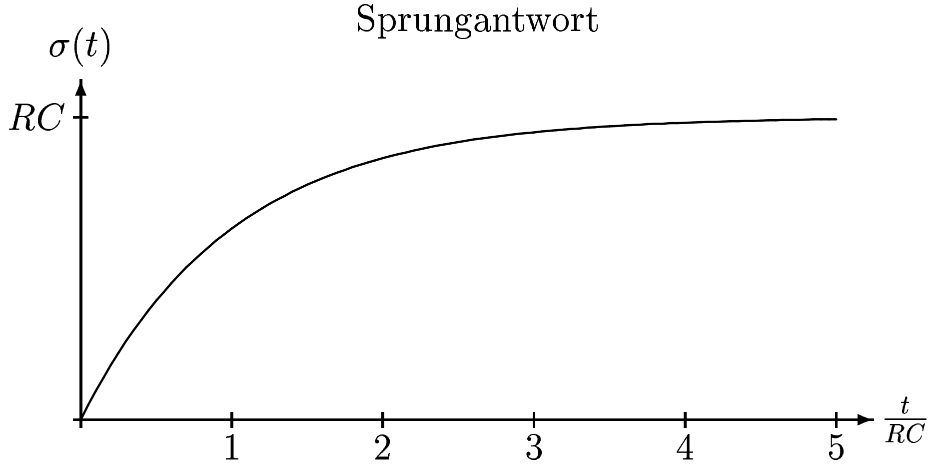
$$\varepsilon(t) = \mathcal{L}^{-1}\left\{\frac{1}{p} \cdot \frac{1}{p + \frac{1}{RC}}\right\}$$

$$= RC\left(1 - \mathrm{e}^{-\frac{t}{RC}}\right)$$

Impulsantwort



Sprungantwort



2. TP 2. Ordnung

$$A(p) = \frac{\frac{1}{LC}}{p^2 + \frac{R}{L}p + \frac{1}{LC}}$$

$$h(t) = \mathcal{L}^{-1}\left\{\frac{\frac{1}{LC}}{p^2 + \frac{R}{L}p + \frac{1}{LC}}\right\}$$

$$\text{Tabelle:} \frac{1}{p^2 + 2ap + b^2} \overset{\mathcal{L}}{\bullet\!\!-\!\!\circ} \begin{cases} \frac{1}{2\sqrt{D}}\left(e^{p_1 t} - e^{p_2 t}\right) & \text{für } D \geq 0 \\ \frac{1}{\sqrt{-D}} e^{-at} \sin(\sqrt{-D}t) & \text{für } D < 0 \end{cases}$$

mit: $D = a^2 - b^2 \qquad p_{1,2} = -a \pm \sqrt{D}$

hier: $a = \frac{R}{2L} \qquad b = \frac{1}{\sqrt{LC}} \qquad D = \frac{R^2}{4L^2} - \frac{1}{LC}$

$$p_{1,2} = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$D \geq 0 \Leftrightarrow \frac{R^2}{4L^2} - \frac{1}{LC} \geq 0 \Leftrightarrow R \geq 2\sqrt{\frac{L}{C}}$$

$$h(t) = \begin{cases} \frac{1}{2\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}}\left(e^{\left(-\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)t} \dots \right. \\ \left. \dots - e^{\left(-\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)t}\right) & , \text{ für } R \geq 2\sqrt{\frac{L}{C}} \\ \frac{1}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} e^{-\frac{R}{2L}t} \cdot \sin\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\, t\right) & , \text{ für } R < 2\sqrt{\frac{L}{C}} \end{cases}$$

$$\sigma(t) = \mathcal{L}^{-1}\left\{\frac{1}{p} \cdot \frac{\frac{1}{LC}}{p^2 + \frac{R}{L}p + \frac{1}{LC}}\right\}$$

entweder Gesetz (Integration im Zeitbereich)

$$= \int_0^t \mathcal{L}^{-1}\left\{\frac{\frac{1}{LC}}{p^2+\frac{R}{L}p+\frac{1}{LC}}\right\}dt$$

$$\int_0^t \mathrm{e}^{ax}dx = \left[\frac{1}{a}\mathrm{e}^{ax}\right]_0^t = \frac{1}{a}\left(\mathrm{e}^{at}-1\right)$$

$$\int_0^t \mathrm{e}^{ax}\sin x dx = \left[\frac{\mathrm{e}^{-ax}}{a^2+1}(a\sin x-\cos x)\right]_0^t = \frac{\mathrm{e}^{-at}}{a^2+1}(a\sin t-\cos t)+\frac{1}{a^2+1}$$

aufwendig, da Fallunterscheidung

oder Tabelle:

$$\frac{1}{p[(p+\beta)^2+\alpha^2]} \overset{\mathcal{L}}{\bullet\!\!-\!\!\circ} \frac{1}{\alpha^2+\beta^2}\left[1-\mathrm{e}^{-\beta t}\left(\cos(\alpha t)+\frac{\beta}{\alpha}\sin(\alpha t)\right)\right]$$

hier: $p^2+2\beta p+\beta^2+\alpha^2 = p^2+\frac{R}{L}p+\frac{1}{LC}$

$\beta = \frac{R}{2L} \quad \beta^2+\alpha^2 = \frac{1}{LC} \Rightarrow \alpha = \sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$

$$\sigma(t) = LC\left[1-\mathrm{e}^{-\frac{R}{2L}}\left(\cos\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\,t\right)\dots\right.\right.$$

$$\left.\left.\dots+\frac{R}{2L\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}}\sin\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\right)\right)\right],$$

für $\frac{1}{LC}-\frac{R^2}{4L^2}\geq 0 \Leftrightarrow R\leq 2\sqrt{\frac{L}{C}}$ wie oben

für $R>2\sqrt{\frac{L}{C}} \Rightarrow \alpha = \sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} = j\sqrt{\frac{R^2}{4L^2}-\frac{1}{LC}}$

und mit: $\cos(jx) = \cosh x = \frac{1}{2}\left(\mathrm{e}^x+\mathrm{e}^{-x}\right)$, sowie mit

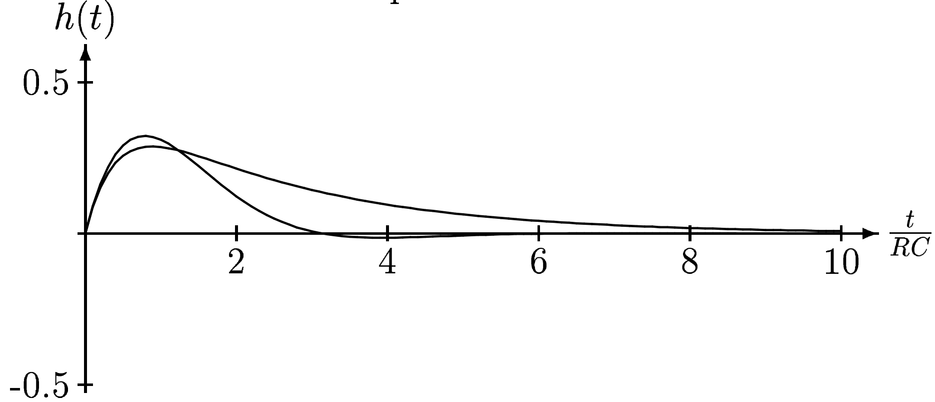
$\sin(jx) = j\sinh x = \frac{j}{2}\left(\mathrm{e}^x-\mathrm{e}^{-x}\right)$

$$\sigma(t) = LC\left[1-\mathrm{e}^{-\frac{R}{2L}}\left(\cosh\left(\sqrt{\frac{R^2}{4L^2}-\frac{1}{LC}}\,t\right)\ldots\right.\right.$$

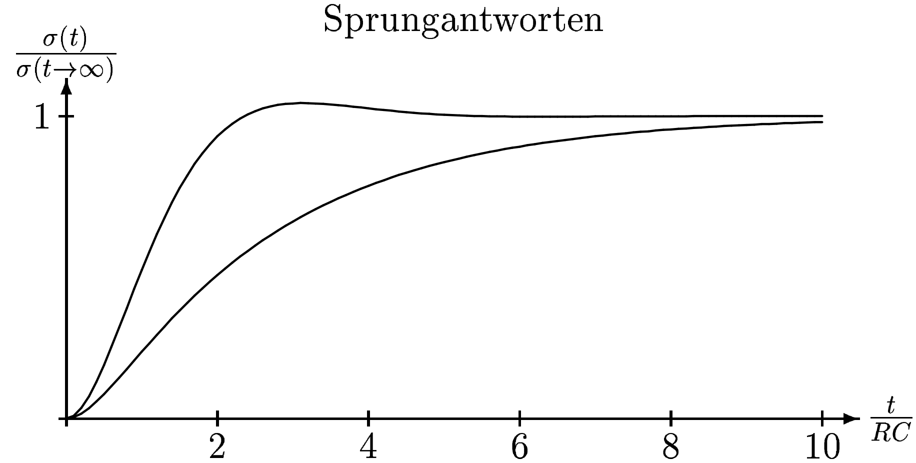
$$\left.\left.\ldots+\frac{R}{2L\sqrt{\frac{R^2}{4L^2}-\frac{1}{LC}}}\sinh\left(\sqrt{\frac{R^2}{4L^2}-\frac{1}{LC}}\,t\right)\right)\right]$$

Zeitverläufe für die Spezialfälle: a) $\alpha = 1$, $\beta = 1$, b) $\alpha = j$, $\beta = 1$.

Impulsantworten



Sprungantworten



Aufgabe 53:

1. LRC-Bandpaß

$$A(p) = \frac{RCp}{p^2 + RCp + LC}$$

$$h(t) = \mathcal{L}^{-1}\left\{\frac{RCp}{p^2 + RCp + LC}\right\}$$

Tabelle: $\frac{p}{(p+\beta)^2 + \alpha^2} \overset{\mathcal{L}}{\bullet\!\!-\!\!\circ}\ \mathrm{e}^{-\beta t}\left(\cos(\alpha t) - \frac{\beta}{\alpha}\sin(\alpha t)\right)$

hier: $p^2 + 2\beta p + \beta^2 + \alpha^2 = p^2 + RCp + LC$

$\beta = \frac{RC}{2} \quad \beta^2 + \alpha^2 = LC \Rightarrow \alpha = \sqrt{LC - \frac{R^2C^2}{4}}$

$$h(t) = \mathrm{e}^{-\frac{RC}{2}t}\left(\cos\left(\sqrt{LC - \frac{R^2C^2}{4}}\,t\right) \ldots\right.$$

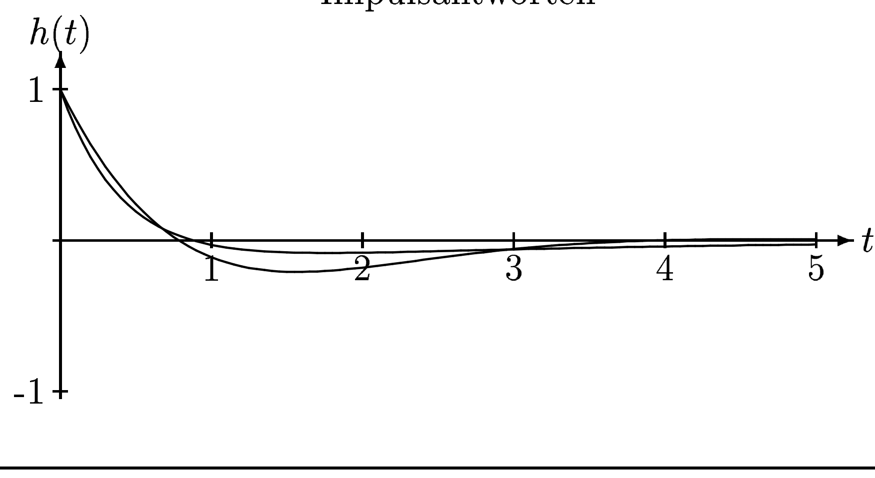
$$\left.\ldots - \frac{RC}{2\sqrt{LC - \frac{R^2C^2}{4}}}\sin\left(\sqrt{LC - \frac{R^2C^2}{4}}\,t\right)\right)$$

$$= \mathrm{e}^{-\frac{RC}{2}t}\left(\cosh\left(\sqrt{\frac{R^2C^2}{4} - LC}\,t\right) \ldots\right.$$

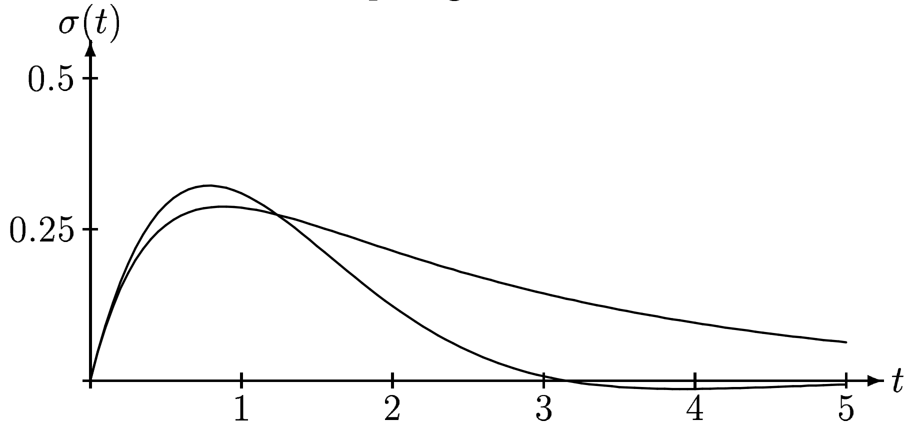
$$\left.\ldots - \frac{RC}{2\sqrt{\frac{R^2C^2}{4} - LC}}\sinh\left(\sqrt{\frac{R^2C^2}{4} - LC}\,t\right)\right)$$

Zeitverläufe für die Spezialfälle: a) $\alpha = 1$, $\beta = 1$, b) $\alpha = j$, $\beta = 1$.

Impulsantworten



Sprungantworten



2. LRC-Bandsperre

$$
\begin{aligned}
A(p) &= \frac{p^2 + LC}{p^2 + RCp + LC} = \frac{p^2 + LC + RCp - RCp}{p^2 + RCp + LC} \\
&= 1 - \frac{RCp}{p^2 + RCp + LC} = 1 - A_{\mathrm{BP}}(p) \\
h(t) &= \mathcal{L}^{-1}\left\{1 - \frac{RCp}{p^2 + RCp + LC}\right\} = \delta(t) - \mathcal{L}^{-1}\left\{\frac{RCp}{p^2 + RCp + LC}\right\} \\
&= \delta(t) - h_{\mathrm{BP}}(t) \\
\sigma(t) &= \varepsilon(t) - \sigma_{\mathrm{BP}}(t)
\end{aligned}
$$

Aufgabe 54:

Dimensionierungsansatz: $f_g \geq 20kHz$ (mit Sicherheitsabstand)
Daraus ergibt sich für die Verzögerung eines Allpasses 2.Ordnung :

$$t_{gr} \leq \frac{2B_1}{\omega_g} = \frac{1,362}{\pi \cdot 20 \cdot 10^3} s = 21,677 \mu s \qquad B_1 \text{ siehe Besseltabelle}$$

$$\text{bzw. } T_{gr} \leq \frac{B_1}{\pi} \qquad \left(A_n(P) = \frac{1 + A_1 P + A_2 P^2}{1 + B_1 P + B_2 P^2} \quad \text{mit } A_1 = -B_1; A_2 = B_2 \right)$$

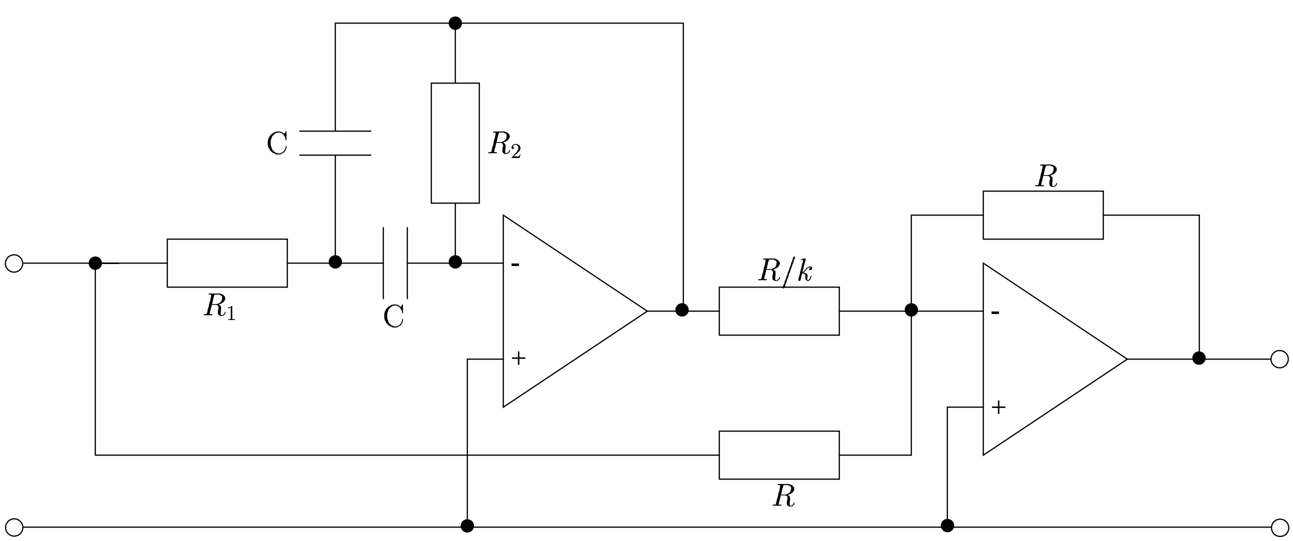
$$= 0,434$$

Es sind also 5 identische Allpässe mit je $20\mu s$ Verzögerungszeit vorzusehen, um $100\mu s$ Gesamtverzögerungszeit zu erreichen.

$$\text{Daher: } f_g \stackrel{!}{=} \frac{B_1}{\pi \cdot 20\mu s} = \frac{1,362}{\pi \cdot 20\mu s} = 21,677kHz$$

wodurch $t_{gr} = 20\mu s$ sich ergibt.

Aber: Ähnlich wie bei Amplitudenfrequenzgängen von Filtern höherer Ordnung kann die Konstanz der Gesamtverzögerungszeit im Durchlaßbereich besser werden, wenn ein Allpaß 10.Ordnung direkt entworfen wird anstatt ihn aus 5 identischen Allpässen zusammenzusetzen. Bestimmung von $A(P)$



Dimensionierung: ($\omega_g = 2\pi \cdot 21,677kHz = 136,2 \cdot 10^3 s^{-1}$)

$$\begin{aligned}
A_1 &= \omega_g(2R_1 - kR_2)C = -B_1 = -1,362 \\
A_2 &= \omega_g^2 R_1 R_2 C^2 = B_2 = 0,618 \\
B_1 &= \omega_g 2R_1 C = 1,362 \\
& \text{Vorgabe: } C = 1nF; \quad R = 10k\Omega \text{ (willkürlich gewählt)} \\
\Rightarrow 1,362 &= \omega_g 2R_1 C \\
\Rightarrow R_1 &= \frac{1,362}{\omega_g \cdot 2 \cdot C} \\
&= \frac{1,362}{0,136 \cdot 10^6 \cdot 2 \cdot 10^{-9} s^{-1} F} = 5,007k\Omega \\
B_2 &= \omega_g^2 R_1 R_2 C^2 \quad \Rightarrow R_2 = \frac{B_2}{\omega_g^2 R_1 C^2} \\
R_2 &= \frac{0,618}{0,018 \cdot 10^{12} \cdot 5 \cdot 10^3 \cdot 10^{-18} s^{-2} \Omega F^2} = 6,867k\Omega \\
B_1 &= \omega_g 2R_1 C \quad \Rightarrow \omega_g C = \frac{B_1}{2R_1} \\
-B_1 &= \omega_g C(2R_1 - kR_2) = \frac{B_1}{2R_1}(2R_1 - kR_2) \\
\Rightarrow -1 &= 1 - \frac{kR_2}{2R_1} \quad \Rightarrow \frac{kR_2}{2R_1} = 2 \quad \Rightarrow k = 4\frac{R_1}{R_2} = 2,875 \\
\Rightarrow \frac{R}{k} &= \frac{10k\Omega}{2,875} = 3,5k\Omega \\
A_N(P) &= \frac{1 - 1,362P + 0,618P^2}{1 + 1,362P + 0,618P^2} \\
A(p) &= A_N\left(\frac{p}{\omega_g}\right) = \frac{1 - 1,362\frac{p}{\omega_g} + 0,618\frac{p^2}{\omega_g^2}}{1 + 1,362\frac{p}{\omega_g} + 0,618\frac{p^2}{\omega_g}} \\
&= \frac{1 - \frac{1,362}{2\pi} f_g p + \frac{0,618}{4\pi^2} f_g^2 p^2}{1 + \frac{1,362}{2\pi} f_g p + \frac{0,618}{4\pi^2} f_g^2 p^2} \\
&= \frac{1 - 4,699 \cdot 10^3 s^{-1} p + 7,356 \cdot 10^6 s^{-2} p^2}{1 + 4,699 \cdot 10^3 s^{-1} p + 7,356 \cdot 10^6 s^{-2} p^2}
\end{aligned}$$