NTS/IW UDE	**Hilfsblatt zur Vorlesung und Übung** Nachrichtentechnische Systeme	FT 1 09/07

Fouriertransformation: Eigenschaften, Abbildungsgesetze

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot \mathrm{e}^{-\mathrm{j}\omega t} \cdot \mathrm{d}t \qquad f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot \mathrm{e}^{\mathrm{j}\omega t} \cdot \mathrm{d}\omega$$

$f_\mathrm{R}(t) = \mathrm{Re}\{f(t)\}$ $\quad f_\mathrm{I}(t) = \mathrm{Im}\{f(t)\}$ $\quad F_\mathrm{R}(\omega) = \mathrm{Re}\{F(\omega)\}$ $\quad F_\mathrm{I}(\omega) = \mathrm{Im}\{F(\omega)\}$
$f(t) = f_\mathrm{g}(t) + f_\mathrm{u}(t)$ mit $f_\mathrm{g}(t) = f_\mathrm{g}(-t)$ und $f_\mathrm{u}(t) = -f_\mathrm{u}(-t)$

a, a_1, a_2 willkürliche Konstanten
b, c, ω_0, t_0 reelle Konstanten

1	$f(t) = f_\mathrm{R}(t) + \mathrm{j} f_\mathrm{I}(t)$	$F_\mathrm{R}(\omega) = \int_{-\infty}^{\infty} [f_\mathrm{R}(t) \cdot \cos(\omega \cdot t) + f_\mathrm{I}(t) \cdot \sin(\omega \cdot t)] \cdot \mathrm{d}t$ $F_\mathrm{I}(\omega) = -\int_{-\infty}^{\infty} [f_\mathrm{R}(t) \cdot \sin(\omega \cdot t) - f_\mathrm{I}(t) \cdot \cos(\omega \cdot t)] \cdot \mathrm{d}t$
2	$f(t)$ reell $\Rightarrow f_\mathrm{I}(t) \equiv 0$	$F_\mathrm{R}(\omega) = F_\mathrm{R}(-\omega)$ gerade Funktion $F_\mathrm{I}(\omega) = -F_\mathrm{I}(-\omega)$ ungerade Funktion $F^*(\omega) = F(-\omega) \quad ; \quad \lvert F(\omega)\rvert = \lvert F(-\omega)\rvert$
3	$f(t)$ imaginär $\Rightarrow f_\mathrm{R}(t) \equiv 0$	$F_\mathrm{R}(\omega) = -F_\mathrm{R}(-\omega)$ ungerade Funktion $F_\mathrm{I}(\omega) = F_\mathrm{R}(-\omega)$ gerade Funktion $F(-\omega) = -F^*(\omega) \quad ; \quad \lvert F(\omega)\rvert = \lvert F(-\omega)\rvert$
4	$f(t)$ reell und gerade $\rightarrow f_\mathrm{R}(t) = f_\mathrm{R}(-t); f_\mathrm{I}(t) \equiv 0$	$F_\mathrm{R}(\omega) = 2 \int_{0}^{\infty} f_\mathrm{R}(t) \cdot \cos(\omega \cdot t) \cdot \mathrm{d}t$ $F_\mathrm{I}(\omega) \equiv 0$
5	$f(t)$ reell und ungerade $\rightarrow f_\mathrm{R}(t) = -f_\mathrm{R}(-t); f_\mathrm{I}(t) \equiv 0$	$F_\mathrm{R}(\omega) \equiv 0$ $F_\mathrm{I}(\omega) = -2 \int_{0}^{\infty} f_\mathrm{R}(t) \cdot \sin(\omega \cdot t) \cdot \mathrm{d}t$
6	$f(t)$ reell $f_\mathrm{g}(t) = \frac{1}{2} \cdot [f(t) + f(-t)]$ $f_\mathrm{u}(t) = \frac{1}{2} \cdot [f(t) - f(-t)]$	$F_\mathrm{R}(\omega) = 2 \int_{0}^{\infty} f_\mathrm{g}(t) \cdot \cos(\omega \cdot t) \cdot \mathrm{d}t$ $F_\mathrm{I}(\omega) = -2 \int_{0}^{\infty} f_\mathrm{u}(t) \cdot \sin(\omega \cdot t) \cdot \mathrm{d}t$ $f_\mathrm{g}(t) = \frac{1}{\pi} \int_{0}^{\infty} F_\mathrm{R}(\omega) \cdot \cos(\omega \cdot t) \cdot \mathrm{d}\omega$ $f_\mathrm{u}(t) = -\frac{1}{\pi} \int_{0}^{\infty} F_\mathrm{I}(\omega) \cdot \sin(\omega \cdot t) \cdot \mathrm{d}\omega$
7	Linearität	$f_1(t) \circ\!\!-\!\!\bullet\, F_1(\omega)$ $f_2(t) \circ\!\!-\!\!\bullet\, F_2(\omega)$ $a_1 \cdot f_1(t) + a_2 \cdot f_2(t) \circ\!\!-\!\!\bullet\, a_1 \cdot F_1(\omega) + a_2 \cdot F_2(\omega)$

Fouriertransformation: Eigenschaften, Abbildungsgesetze		
8	Ähnlichkeit, Maßstabsänderung	$f(t) \circ\!\!-\!\!\bullet F(\omega)$ $f(b\cdot t) \circ\!\!-\!\!\bullet \frac{1}{\|b\|}\cdot F\left(\frac{\omega}{b}\right)$ $F(c\cdot\omega) \bullet\!\!-\!\!\circ \frac{1}{\|c\|}\cdot f\left(\frac{t}{c}\right)$
9	Verschiebung	$f(t-t_0)$ $f(t-t_0) \circ\!\!-\!\!\bullet F(\omega)\cdot \mathrm{e}^{-\mathrm{j}\omega t_0}$ $F(\omega-\omega_0)$ $F(\omega-\omega_0) \bullet\!\!-\!\!\circ f(t)\cdot \mathrm{e}^{+\mathrm{j}\omega_0 t}$
10	Differentiation im Zeitbereich	$f(t)$ n-mal differenzierbar und $F(\omega)$ existiert und $\lim\limits_{t\to\pm\infty} f^{(\nu)}(t)=0$ für $\nu=0,1,\ldots,(n-1)$ $\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n} \circ\!\!-\!\!\bullet (\mathrm{j}\omega)^n\cdot F(\omega)$
11	Differentiation im Frequenzbereich	$F^{(n)}(\omega)$ existiert $\frac{\mathrm{d}^n F(\omega)}{\mathrm{d}\omega^n} \bullet\!\!-\!\!\circ (-\mathrm{j}t)^n\cdot f(t)$
12	Integration im Zeitbereich	Wenn $g(t)=\int_{-\infty}^{t} f(\tau)\cdot\mathrm{d}\tau$ absolut integrierbar $\int_{-\infty}^{t} f(\tau)\cdot\mathrm{d}\tau \circ\!\!-\!\!\bullet \frac{1}{\mathrm{j}\omega}\cdot F(\omega)$ für $F(0)=0$ $\int_{-\infty}^{t} f(\tau)\cdot\mathrm{d}\tau \circ\!\!-\!\!\bullet \frac{1}{\mathrm{j}\omega}\cdot F(\omega)+\pi\cdot F(0)\cdot\delta(\omega)$ für $F(0)\neq 0$
13	Multiplikation und Faltung	$f_1(t)*f_2(t)=\int_{-\infty}^{\infty} f_1(\tau)\cdot f_2(t-\tau)\cdot\mathrm{d}\tau$ $f_1(t)*f_2(t) \circ\!\!-\!\!\bullet F_1(\omega)\cdot F_2(\omega)$ $f_1(t)\cdot f_2(t) \circ\!\!-\!\!\bullet \frac{1}{2\pi}\cdot F_1(\omega)*F_2(\omega)$
14	Parsevals Theorem	$f_1(t), f_2(t)$ absolut als auch quadratisch integrierbar $\int_{-\infty}^{\infty} f_1(t)\cdot f_2(t)\cdot\mathrm{d}t = \frac{1}{2\pi}\cdot\int_{-\infty}^{\infty} F_1(-\omega)\cdot F_2(\omega)\cdot\mathrm{d}\omega$ $f_1(t)$ reell $\rightarrow$ $\int_{-\infty}^{\infty} f_1^2(t)\cdot\mathrm{d}t = \frac{1}{2\pi}\cdot\int_{-\infty}^{\infty} \|F_1(\omega)\|^2\cdot\mathrm{d}\omega$
15	Symmetrie	$f(t) \circ\!\!-\!\!\bullet F(\omega)$ dann gilt : $F\left(-\frac{t}{\gamma}\right) \circ\!\!-\!\!\bullet 2\pi\|\gamma\|\cdot f(\omega\cdot\gamma)$ beide bekannt γ =reeller Skalierungsfaktor

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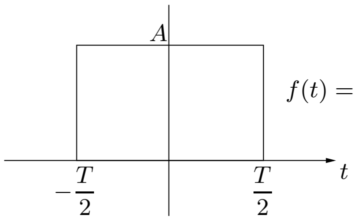
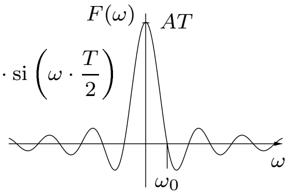
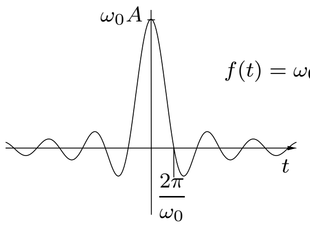
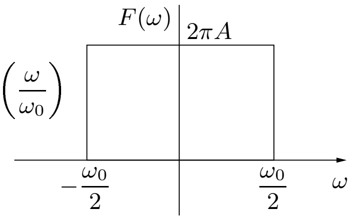
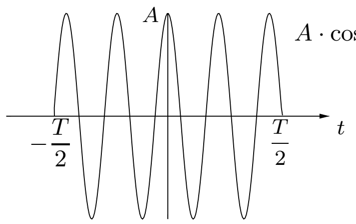
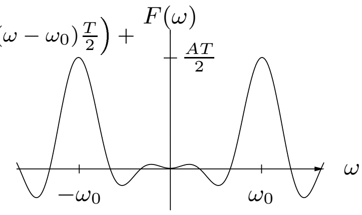
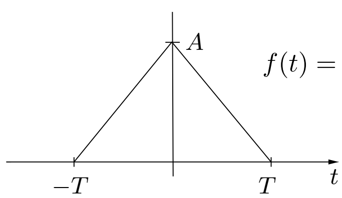
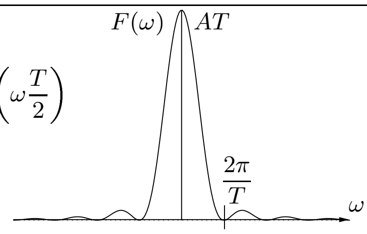
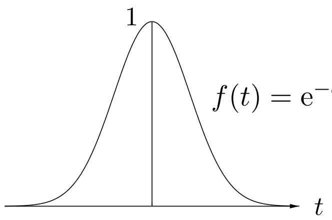
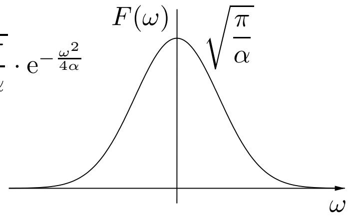
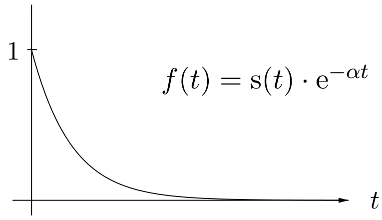
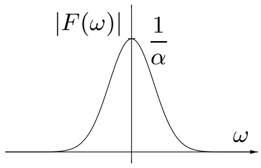
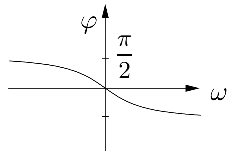
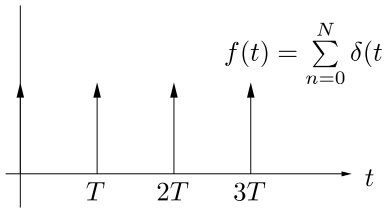
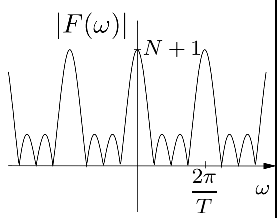
Fourier-Korrespondenzen

	$f(t) \circ\!\!-\!\!\bullet F(\omega)$	
1	 $f(t) = A \cdot \text{rect}\left(\frac{t}{T}\right)$	$\omega_0 = \frac{2 \cdot \pi}{T}$ $F(\omega) = A \cdot T \cdot \text{si}\left(\omega \cdot \frac{T}{2}\right)$ 
2	 $f(t) = \omega_0 A \cdot \text{si}\left(\frac{\omega_0}{2} \cdot t\right)$	$F(\omega) = 2\pi A \cdot \text{rect}\left(\frac{\omega}{\omega_0}\right)$ 
3	 $A \cdot \cos(\omega_0 t) \cdot \text{rect}\left(\frac{t}{T}\right)$	$F(\omega) = \frac{AT}{2}\left[\text{si}\left((\omega-\omega_0)\frac{T}{2}\right) + \text{si}\left((\omega+\omega_0)\frac{T}{2}\right)\right]$ 
4	 $f(t) = A \cdot \Delta\left(\frac{t}{T}\right)$	$F(\omega) = AT \cdot \text{si}^2\left(\omega\frac{T}{2}\right)$ 
5	 $f(t) = \mathrm{e}^{-\alpha t^2}$	$F(\omega) = \sqrt{\frac{\pi}{\alpha}} \cdot \mathrm{e}^{-\frac{\omega^2}{4\alpha}}$ 
6	 $f(t) = \mathrm{s}(t) \cdot \mathrm{e}^{-\alpha t}$	$F(\omega) = \frac{1}{\alpha + \mathrm{j}\omega}$  
7	 $f(t) = \sum_{n=0}^{N} \delta(t - nT)$	$F(\omega) = \sum_{n=0}^{N} \mathrm{e}^{-\mathrm{j}n\omega T} = \mathrm{e}^{-\mathrm{j}N\omega\frac{T}{2}} \cdot \frac{\sin\left((N+1)\cdot\omega\frac{T}{2}\right)}{\sin\left(\omega\frac{T}{2}\right)}$ 

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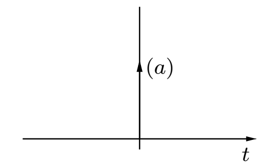
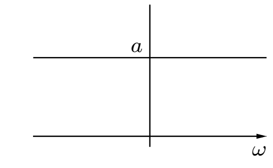
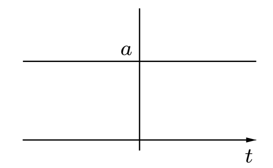
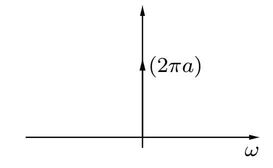
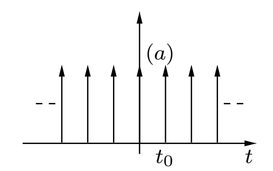
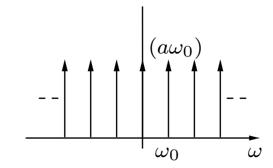
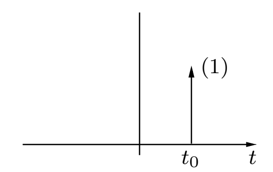
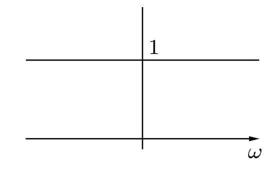
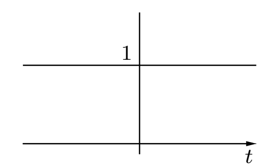
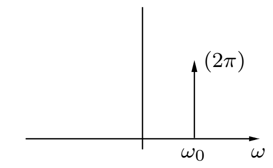
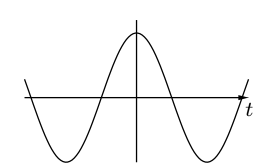
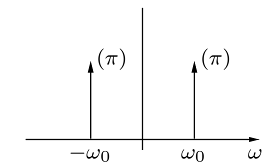
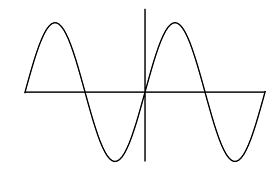
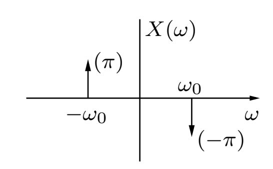
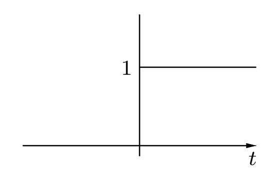
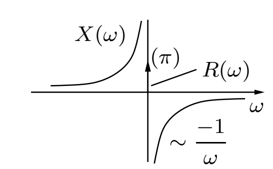
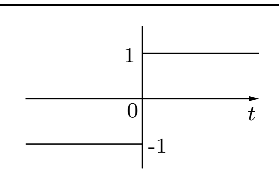
Fourier-Korrespondenzen

	$f(t)$		$F(\omega)$	$\lvert F(\omega)\rvert$
9		$a \cdot \delta(t)$	a	
10		a	$2\pi a \cdot \delta(\omega)$	
11		$a \cdot \sum_{n=-\infty}^{\infty} \delta(t - nt_0)$	$\omega_0 = \frac{2\pi}{t_0}$ $a\omega_0 \cdot \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	
12		$\delta(t - t_0)$	$\mathrm{e}^{-\mathrm{j}\omega t_0}$	
13		$\mathrm{e}^{\mathrm{j}\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
14		$\cos(\omega_0 t)$	$\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	
15		$\sin(\omega_0 t)$	$\mathrm{j}\pi\delta(\omega + \omega_0) -$ $\mathrm{j}\pi\delta(\omega - \omega_0)$	
16		$\mathrm{s}(t)$ $\mathrm{s}(0) \overset{!}{=} \frac{1}{2}$	$\frac{1}{\mathrm{j}\omega} + \pi\delta(\omega)$	
17		$\mathrm{sgn}(t)$	$\frac{2}{\mathrm{j}\omega}$