Network Theory 1
Analoge Netzwerke

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Chapter 3

Realization of active RC two-port networks

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3.1 Preface

• The active RC two-port led to important developments and/or realization of high-performance operational amplifiers in integrated technology

• Here are some main reasons of this important development:

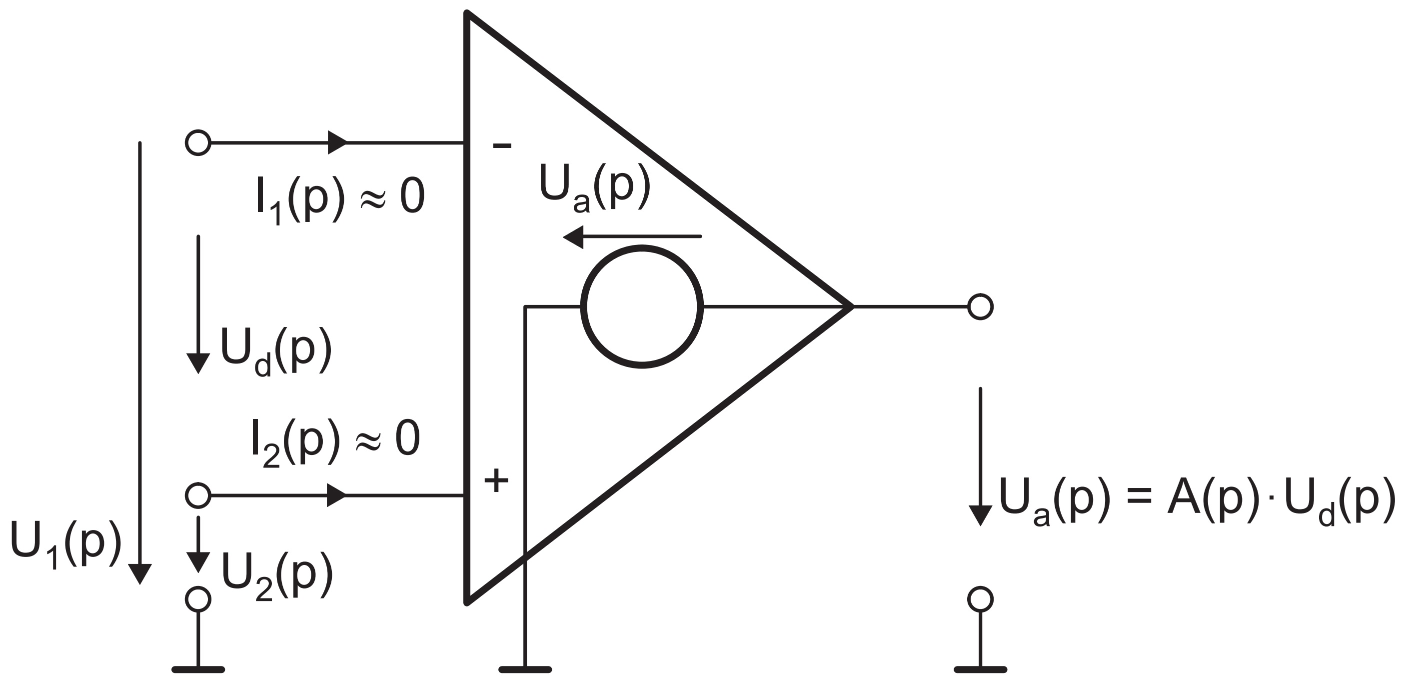
1- Active RC two-port realize system functions with practically arbitrary pole zero configuration in avoidance of coils.

2- OHM's resistances and special passive capacities as well as the necessary amplifiers can be produced very accuractely in integrated technology and thereby fulfilling the conditions for miniaturized complex systems.

3- The negative impedance converter and gyrator realizable with OHM's resistances and operational amplifiers extend the possibilities of realizing system functions.

3.2 The operational amplifier

A simple linear model for the description of the most important characteristics of real operational amplifiers looks like this symbol:



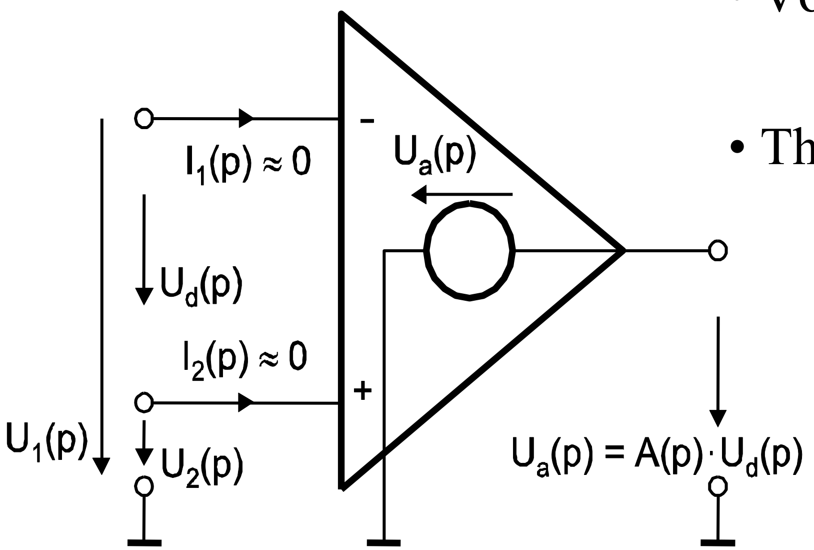
3.2 The operational amplifier

The model represents an idealization of the operational amplifier (op-amp) because:

- The internal amplifier DC-offset source and noise sources are neglected
- The internal resistance R_i of the internal voltage-controlled voltage supply is considered as 0 (usually from 0.01Ω .. 5Ω)
- The difference impedance Z_d is set to ∞ (usually $Z_d = 100k\Omega - 10M\Omega$)
- The common mode input impedance $Z_G(p)$ (usually $2,5M\Omega - 10G\Omega$), which lies between E_1, E_2 and the ground, is set also to ∞. Thus, there is no current flowing into E_1, E_2 when Z_d is set to infinity.
- Any nonlinear $u_a - u_d$ characteristic of the op-amp is neglected.

3.2 The operational amplifier

Under these conditions, one obtains the followings results:



Simple linear model of a real operational amplifier

- Voltage difference: $U_d(p) = U_1(p) - U_2(p)$

- The output voltage:

$$U_a(p) = A(p)\ U_d(p)$$

- The open-loop amplification $A(p)$:

$$A(p) = \frac{A_0 \cdot \omega_g}{p + \omega_g} \quad \text{where } A_0 > 0$$

3.2 The operational amplifier

Some discussions:

- The DC voltage open-loop gain A_0 has the value $10^3 \ldots 10^7$
- The 3dB cut-off frequency ω_g has values 1Hz ... 2kHz
- The transit frequency f_T is in the range 1 Mhz ... 200 MHz
- The operational amplifier output voltage is finite
- The open-loop voltage gain $A_0 >> 1$ leads to negligable differential input voltage (due to the large value of the output)

- Under normal operating conditions, practically one can consider with

$$U_1(p) \approx U_2(p) \quad \text{for} \quad A_0 >> 1$$

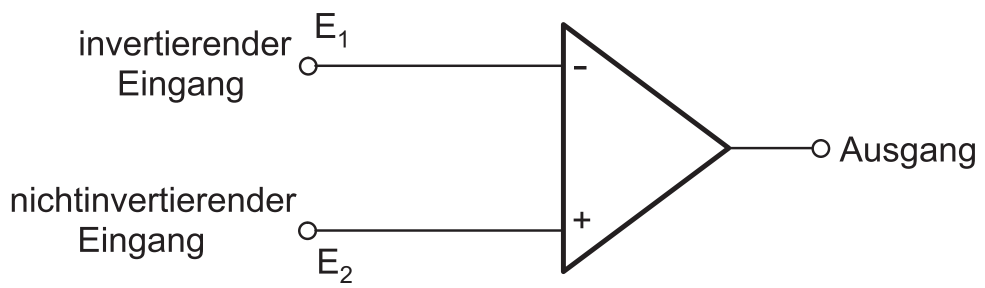
(due to feedback and large open-loop gain)

3.2 The operational amplifier

Using the linear operation amplifier model shown and $A_0 \rightarrow \infty$, the model of the ideal operation amplifier is obtained with the following characterizing relations:

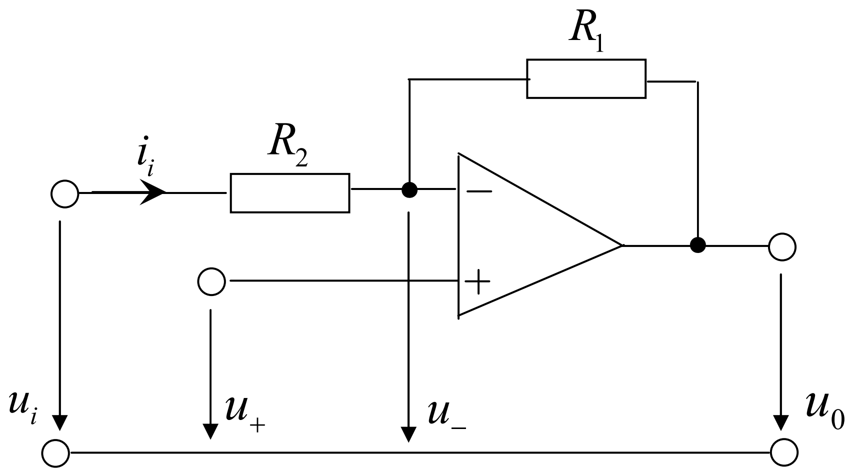
- $U_1(p) = U_2(p) \quad \text{for } A_0 \rightarrow \infty$
- $I_1(p) = I_2(p) = 0$

where the output voltage exhibits finite values and is offset-free and noise-free.



Symbol for an ideal operational amplifier

3.2 The operational amplifier



Basic circuit of an operational amplifier

$$u_0 = A_0(u_+ - u_-)$$

$$u_i = i_i(R_1 + R_2) + u_0$$

$$u_- = u_i - i_i \cdot R_2$$

These relations can be solved for the output by eliminating u_- and i_i:

$$u_0 = \frac{u_+(1+k) - u_i k}{1 + (1+k)/A_0} \quad \text{with} \quad k = \frac{R_1}{R_2}$$

Thus it results for $A_0 \to \infty$:

$$\lim_{A_0 \to \infty} u_0 = u_+(1+k) - u_i k$$

$$= k(u_+ - u_i) + u_+$$

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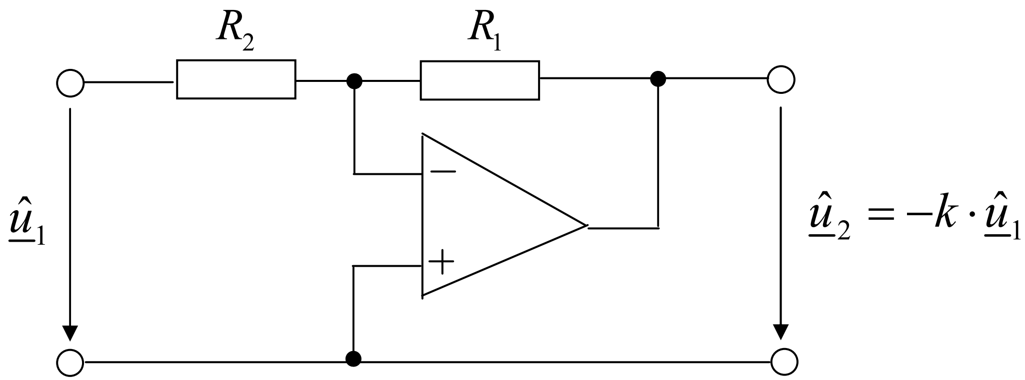
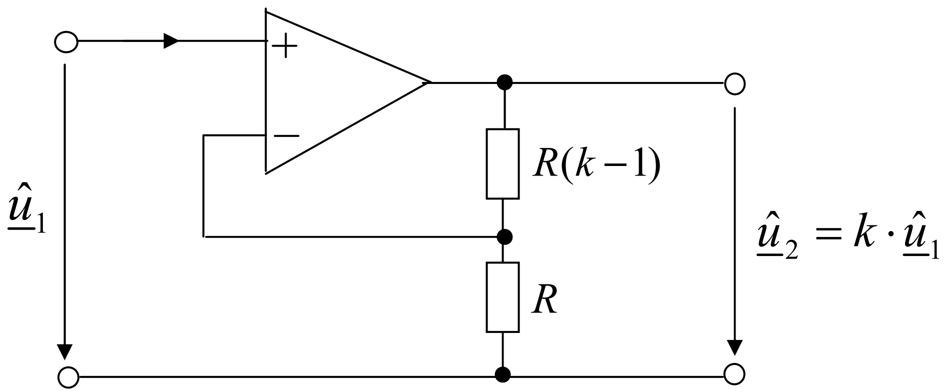
3.2 The operational amplifier

First applications:



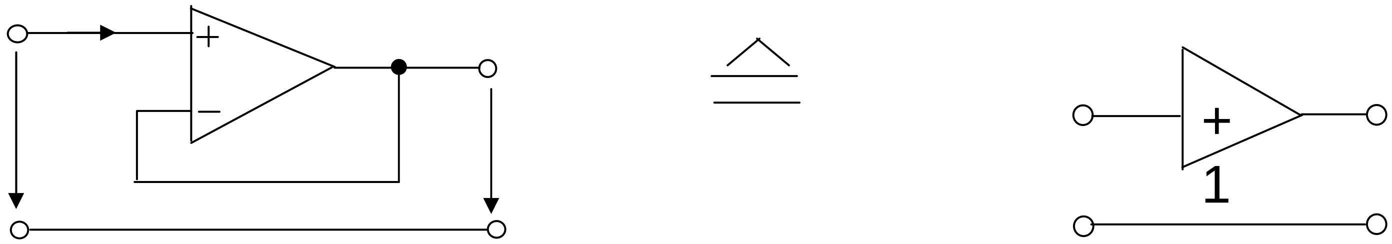
An amplifier with easily settable negative gain k



An amplifier with easily settable positive gain

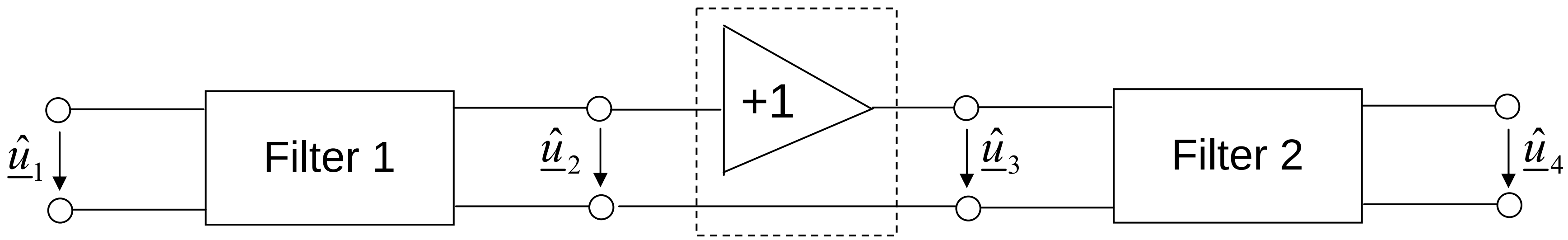
3.2 The operational amplifier

A special case but important case results for $k = 1 \rightarrow R_1 = 0$ and $i_- = 0$:



Circuit of a voltage follower and its simplified symbol

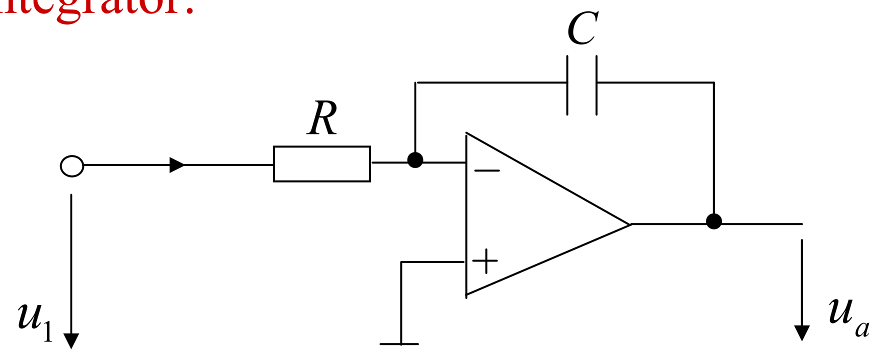
Decoupling circuit



Use of a decoupling circuit in between filter modules

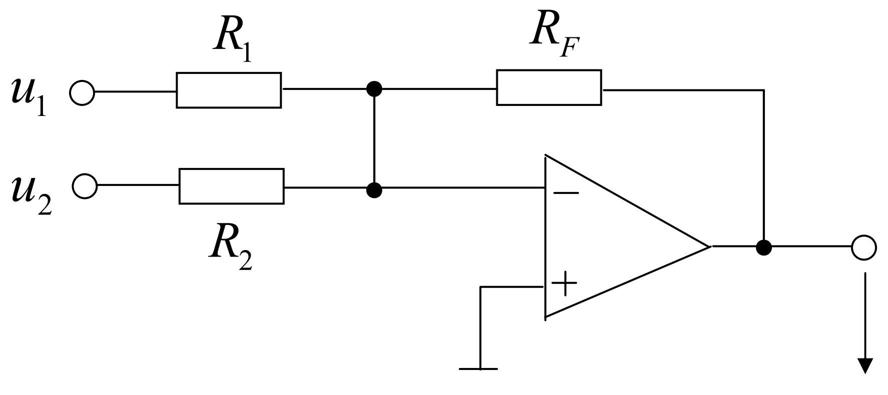
3.2 The operational amplifier

Ideal integrator:



$$H_L(p) = A(p) = \frac{-\,1/pC}{R}$$

Ideal adder:



$$u_a = -\frac{R_F}{R_1}\cdot u_1 - \frac{R_F}{R_2}\cdot u_2$$

3.3 Active RC two-port with infinitely amplifying Op-amp and individual feedback

3.3.1 Structure and Properties

Structure of an active RC two-port with ideal op-amp

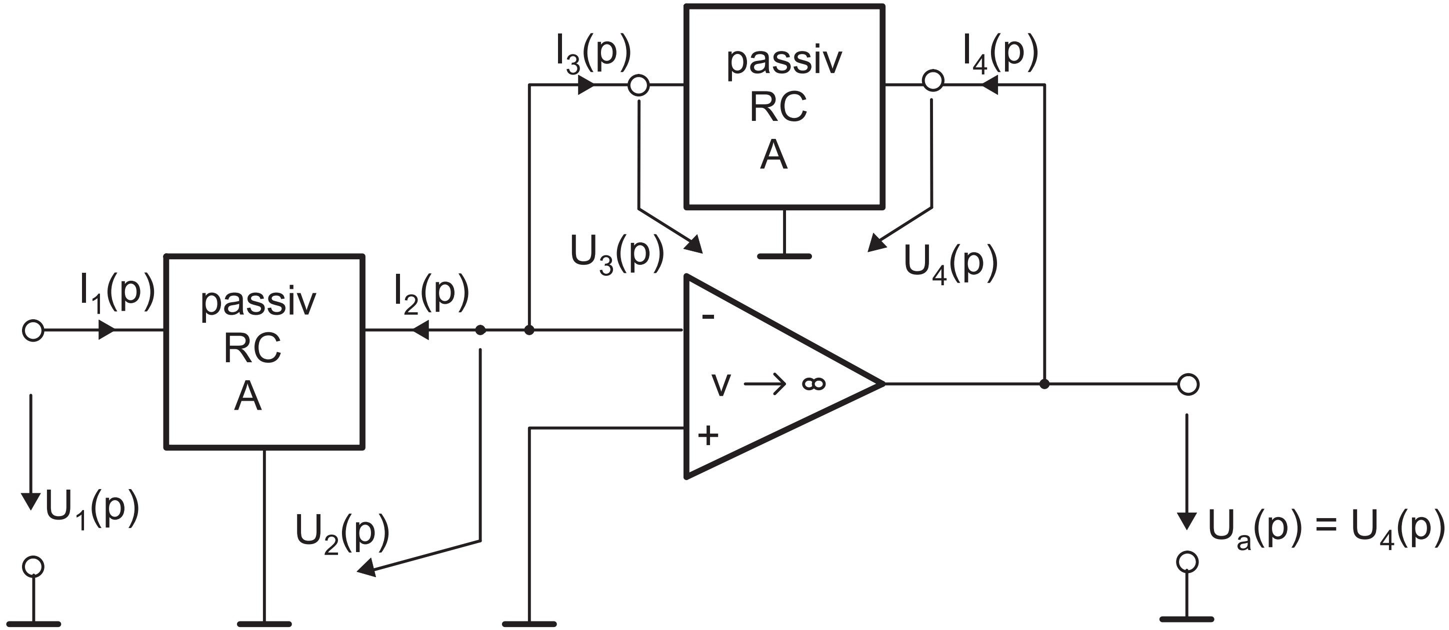


Fig. 3.3.1: Infinite gain single feedback circuit

3.3.1 Structure and Properties

It applies to the two-port A (being a normal passive two-port device):

$$I_1(p) = Y_{11A}(p) \cdot U_1(p) + Y_{12A}(p) \cdot U_2(p)$$

$$I_2(p) = Y_{21A}(p) \cdot U_1(p) + Y_{22A}(p) \cdot U_2(p)$$

Similarly for two-port B:

$$I_3(p) = Y_{11B}(p) \cdot U_3(p) + Y_{12B}(p) \cdot U_4(p)$$

$$I_4(p) = Y_{21B}(p) \cdot U_3(p) + Y_{22B}(p) \cdot U_4(p)$$

3.3.1 Structure and Properties

As shown in figure 3.3.1, one observes the following:

$$U_2(p) = U_3(p) = 0 \quad \text{(because input of opamp is virtually grounded)}$$

under this condition, one gets:

$$I_2(p) = Y_{21A}(p) \cdot U_1(p)$$

and $$I_3(p) = Y_{12B}(p) \cdot U_4(p)$$

Furthermore, it is directly clear from figure 3.3.1 that: $I_2(p) = -I_3(p)$

This gives:

System function of the network

$$Y_{21A}(p) \cdot U_1(p) = -Y_{12B}(p) \cdot U_4(p)$$

$$\Leftrightarrow U_4(p) = -\frac{Y_{21A}(p)}{Y_{12B}(p)} \cdot U_1(p) = H_L(p) \cdot U_1(p) \quad \text{where} \quad H_L(p) = -\frac{Y_{21A}(p)}{Y_{12B}(p)}$$

3.3.1 Structure and Properties

The input admittance of the network can be determined as:

$$Y_e(p) = \left.\frac{I_1(p)}{U_1(p)}\right|_{U_2(p)=0} = Y_{11A}(p)$$

<u>Pole and zero determination:</u>

If one expresses $Y_{21\,A}(p)$ and $Y_{12\,B}(p)$ in terms of fractions, then:

- Poles of $H_L(p)$ correspond to poles of $Y_{21\,A}(p)$ together with zeros of $Y_{12\,B}(p)$
- Zeros of $Y_{21\,A}(p)$ together with poles of $Y_{12\,B}(p)$ correspond to zeros of $H_L(p)$

→ From this, the realization of RC network A and B can be derived based on the system function using pole and zero specifications!

3.3.2 The active 2nd order RC low-pass

<u>Typical application example:</u>

A stable second order low-pass with negative amplification for DC has the system function of

$$H_L(p) = -\frac{H_0 \cdot \omega_g^2}{p^2 + b \cdot \omega_g p + d \cdot \omega_g^2} = -\frac{H_0}{p^2/\omega_g^2 + b \cdot p/\omega_g + d}$$

with $0 < b < 2$ in case of $d = 1$

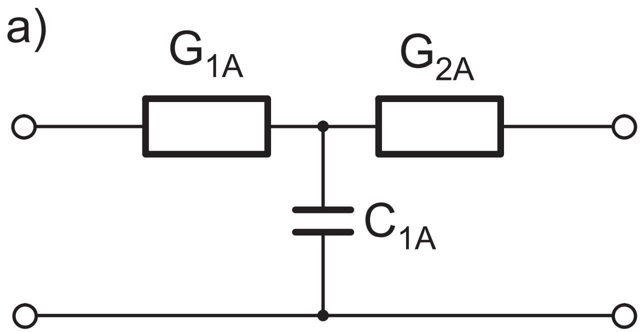
and $H_0 = H_L(0)$ (DC voltage gain)

Under the conditions given above, the system exhibits accordingly the conjugated complex pair of poles for $b < 2$ in case of $d = 1$:

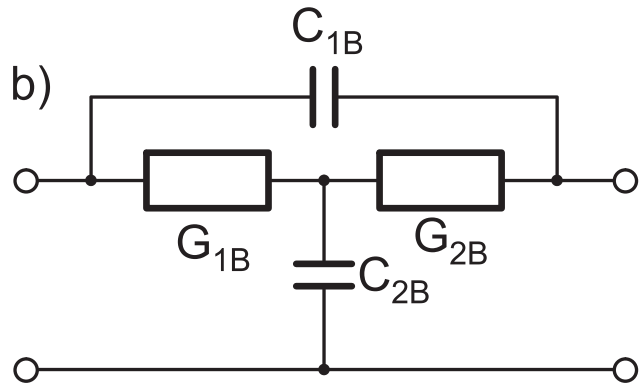
$$p_{\infty 1,2} = -\frac{b}{2}\omega_g \pm j\omega_g\sqrt{1-\frac{b^2}{4}}$$

3.3.2 The active 2nd order RC low-pass

Passive RC two-ports for the realization of an active RC low-pass:

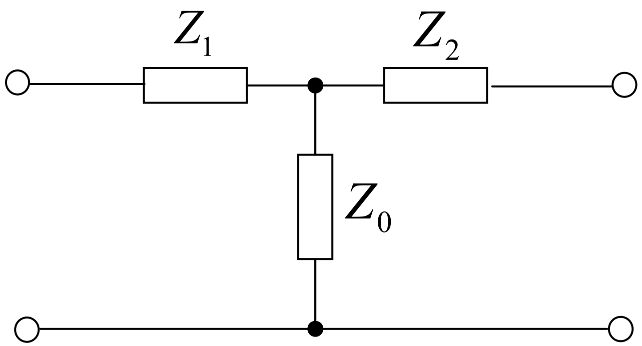


a) RC Two-port A (T-type)



b) RC Two-port B (Bridged T-type)

For any T-circuit like those shown above it holds:



$$\overleftrightarrow{Z} = \begin{pmatrix} Z_1 + Z_0 & Z_0 \\ Z_0 & Z_2 + Z_0 \end{pmatrix}$$

3.3.2 The active 2nd order RC low-pass

At first Y_{12} is determined using Z-parameters as follows:

$$Y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-Z_0}{Z_1 Z_2 + Z_1 Z_0 + Z_0 Z_2} \quad \text{due to} \quad \Delta Z = (Z_1 + Z_0)(Z_2 + Z_0) - (Z_0)^2$$

For the <u>first part</u> of two-port B (T-circuit without C_{1B} denoted by index „12T“) it holds:

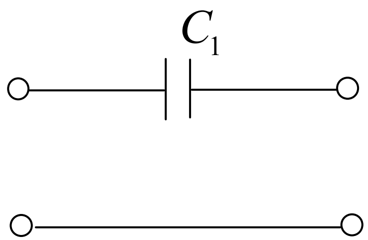
$$Z_1 = \frac{1}{G_{1B}} \qquad Z_2 = \frac{1}{G_{2B}} \qquad Z_0 = \frac{1}{pC_{2B}} \quad \Rightarrow$$

$$Y_{12T} = -\frac{1}{pC_{2B}} \cdot \frac{1}{1/G_{1B}G_{2B} + 1/G_{1B}pC_{2B} + 1/G_{2B}pC_{2B}}$$

$$= -\frac{1}{\frac{pC_{2B}}{G_{1B}G_{2B}} + \frac{1}{G_{1B}} + \frac{1}{G_{2B}}} = -\frac{G_{1B}G_{2B}}{pC_{2B} + G_{1B} + G_{2B}}$$

3.3.2 The active 2nd order RC low-pass

For the <u>second part</u> of two-port B (π-circuit), it holds:



$$Y_{12\pi} = -Y_0 = -pC_1$$

Thus it results due to:

$$Y_{12B}(p) = Y_{12T} + Y_{12\pi}$$

$$Y_{12B}(p) = -pC_{1B} - \frac{G_{1B}G_{2B}}{pC_{2B} + G_{1B} + G_{2B}} = -\frac{pC_{1B} \cdot (pC_{2B} + G_{1B} + G_{2B}) + G_{1B}G_{2B}}{pC_{2B} + G_{1B} + G_{2B}}$$

$$= -C_{1B} \frac{p^2 C_{2B} + p \cdot (G_{1B} + G_{2B}) + G_{1B}G_{2B} / C_{1B}}{pC_{2B} + G_{1B} + G_{2B}}$$

$$= -C_{1B} \frac{p^2 + p\dfrac{G_{1B} + G_{2B}}{C_{2B}} + \dfrac{G_{1B}G_{2B}}{C_{1B}C_{2B}}}{p + \dfrac{G_{1B} + G_{2B}}{C_{2B}}}$$

3.3.2 The active 2nd order RC low-pass

For the T-circuit of network A, which is similar to first part of network B, it holds:

$$Y_{21A}(p) = -\frac{G_{1A}G_{2A}}{pC_{1A} + G_{1A} + G_{2A}} = -\frac{\dfrac{G_{1A}G_{2A}}{C_{1A}}}{p + \dfrac{G_{1A} + G_{2A}}{C_{1A}}}$$

The system function of the filter then becomes:

$$H_L(p) = -\frac{Y_{21A}(p)}{Y_{12B}(p)} = -\frac{\dfrac{G_{1A}G_{2A}}{C_{1A}} \cdot \dfrac{1}{C_{1B}} \left(p + \dfrac{G_{1B} + G_{2B}}{C_{2B}}\right)}{\left(p + \dfrac{G_{1A} + G_{2A}}{C_{1A}}\right) \cdot \left(p^2 + \dfrac{G_{1B} + G_{2B}}{C_{2B}} p + \dfrac{G_{1B} \cdot G_{2B}}{C_{1B} \cdot C_{2B}}\right)}$$

A conjugate-complex pair of zeros in addition to a simple pole on the negative σ-axis can be realized by suitable choice of conductances G_{1B}, G_{2B} and capacities C_{1B}, C_{2B}.

3.3.2 The active 2nd order RC low-pass

The wanted system function in this example has no zero, so that the admittance function of RC two-port A must have an appropriated pole compensating the zero.

One pole and one zero of the system function can be cancelled from the fraction given above under the condition that:

$$\frac{G_{1A}+G_{2A}}{C_{1A}}=\frac{G_{1B}+G_{2B}}{C_{2B}} \quad (0)$$

Thus that the system function will be:

$$H_L(p)=-\frac{\dfrac{G_{1A}\cdot G_{2A}}{C_{1A}\cdot C_{1B}}}{p^2+\dfrac{G_{1B}+G_{2B}}{C_{2B}}\cdot p+\dfrac{G_{1B}\cdot G_{2B}}{C_{1B}\cdot C_{2B}}}$$

3.3.2 The active 2nd order RC low-pass

Comparison of coefficients with the desired system function gives:

$$\frac{G_{1A}\cdot G_{2A}}{C_{1A}\cdot C_{1B}} = H_0\cdot\omega_g^2 \qquad \frac{G_{1B}+G_{2B}}{C_{2B}} = b\cdot\omega_g \qquad \frac{G_{1B}\cdot G_{2B}}{C_{1B}\cdot C_{2B}} = \omega_g^2$$

These 3 relations together with the relation due to the zero compensation condition (0) is not enough to define all 7 unknown component values.

So another 3 additional relations/conditions are needed. Possibilities for such relations are e.g.:

$$C_{1A} = \frac{AG_w}{\omega_g}\ (10) \qquad C_{2B} = \frac{BG_w}{\omega_g}\ (11) \qquad \mathrm{G}_{1\mathrm{B}} = kG_w\ (12)$$

with A, B, k and G_w set to suitable values.

3.3.2 The active 2nd order RC low-pass

With this 3 additional relations the previous 4 relations can be rewitten like this to solve the system of 7 equations:

$$(1)\quad \frac{G_{1A}\cdot G_{2A}}{AG_w\cdot C_{1B}} = H_0\omega_g$$

$$(2)\quad \frac{kG_w + G_{2B}}{BG_w} = b \quad\Rightarrow\quad G_{2B} = BG_w b - kG_W = G_w(Bb-k)$$

$$(3)\quad \frac{kG_w G_{2B}}{C_{1B}BG_w} = \omega_g \quad\Rightarrow\quad C_{1B} = \frac{kG_w G_{2B}}{\omega_g BG_w}$$

$$= \frac{k}{\omega_g B}G_{2B} = \frac{G_w k}{\omega_g B}(Bb-k)$$

$$(4)\quad \omega_g\frac{G_{1A}+G_{2A}}{AG_w} = \frac{kG_w+G_{2B}}{BG_w}\omega_g \quad\Rightarrow\quad \frac{G_{1A}+G_{2A}}{A} = \frac{kG_w+G_{2B}}{B}$$

3.3.2 The active 2nd order RC low-pass

(1) gives with solved C_{1B} :

$$\frac{G_{1A} \cdot G_{2A}}{AG_w \cdot \frac{G_w k}{\omega_g B}(Bb-k)} = \frac{G_{1A} \cdot G_{2A} \cdot B\omega_g}{G_w^2 Ak(Bb-k)} = H_0 \omega_g \quad (5)$$

(4) gives with solved C_{1B} :

$$\frac{G_{1A} + G_{2A}}{A} = \frac{kG_w + G_w(Bb-k)}{B} = \frac{G_w Bb}{B} = G_w b$$

$$\Rightarrow \quad G_{1A} + G_{2A} = AG_w b \ \text{ or } \ G_{2A} = AG_w b - G_{1A} \quad (6)$$

3.3.2 The active 2nd order RC low-pass

(6) in (5) then gives:

$$\frac{G_{1A}(AG_w b - G_{1A})B}{G_w^2 Ak(Bb-k)} = H_0$$

$$\Rightarrow AG_w bB \cdot G_{1A} - B \cdot G_{1A}^2 = G_w^2 Ak(Bb-k)H_0$$

$$G_{1A}^2 - G_w Ab \cdot G_{1A} + G_w^2 Ak(b - k/B)H_0 = 0$$

$$\Rightarrow G_{1A_{1,2}} = G_w \frac{Ab}{2} \pm G_w \sqrt{\frac{A^2 b^2}{4} - Ak(b - k/B)H_0}$$

$$= G_w \frac{Ab}{2}\left(1 \pm \sqrt{1 - \frac{4k(b - k/B)H_0}{Ab^2}}\right) = G_w \frac{Ab}{2}\left(1 \pm \sqrt{1 - \frac{4k(bB-k)H_0}{ABb^2}}\right) \quad (7)$$

Parameters/sign must be chosen such that G_{1A} is positive and real!

$$\Rightarrow ABb^2 \le 4k(bB-k)H_0$$

3.3.2 The active 2nd order RC low-pass

An example of the design of an active RC low-pass

An active RC low-pass 2nd Order should show the following characteristics:

- DC voltage open-loop gain $H_0 = 10 \;\; (\hat{=}\, 20dB)$
- Low-pass cut-off frequency $f_g = 1 \;\; kHz$
- Constants $b = \sqrt{2}$ and $d = 1$

  (these values are required for a BUTTERWORTH low-pass)

First, the following values are chosen arbitrarily:

- Constants $k = 1 \;\; A = 2 \;\;\; B = 24$
- $R_w = 10 \;\; k\Omega$

3.3.2 The active 2nd order RC low-pass

Then one obtains:

$C_{1A} = 318\ nF$ from (10)

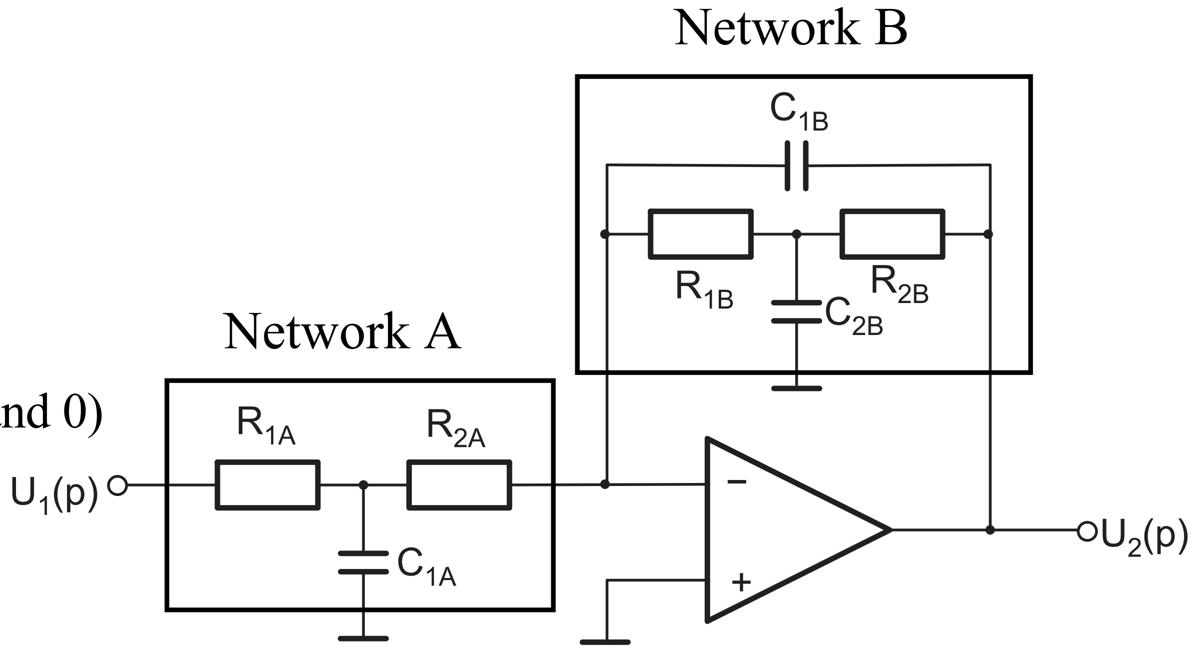
$C_{2B} = 38{,}4\ nF$ from (11)

$R_{1B} = 10\ k\Omega$ from (12)

$R_{1A} = R_{2A} = 707\ \Omega$ from (7 and 0)

$R_{2B} = 4{,}142\ k\Omega$ from (2)

$C_{1B} = 15{,}9\ nF$ from (3)



Active RC low-pass 2nd Order (BUTTERWORTH)

Another, easier method is to directly specify all 3 capacitor values and then solve the equations given in S. 24.

For low-passes of 2nd order specialized circuits are known which simplify the design process even more.

3.3.3 The active 2nd order RC high-pass

The general stable second order high-pass with negative amplification for highest frequencies has the system function:

$$H_L(p) = -\frac{H_0 \cdot p^2}{p^2 + b \cdot \omega_g \cdot p + d \cdot \omega_g^2} \quad \text{with } \omega_g > 0, H_0 > 0, \text{ real and } 0 < b < 2$$

$$\text{for } d = 1 \text{ and } H_0 = \lim_{\omega \to \infty} \{-H_L(j\omega)\}$$

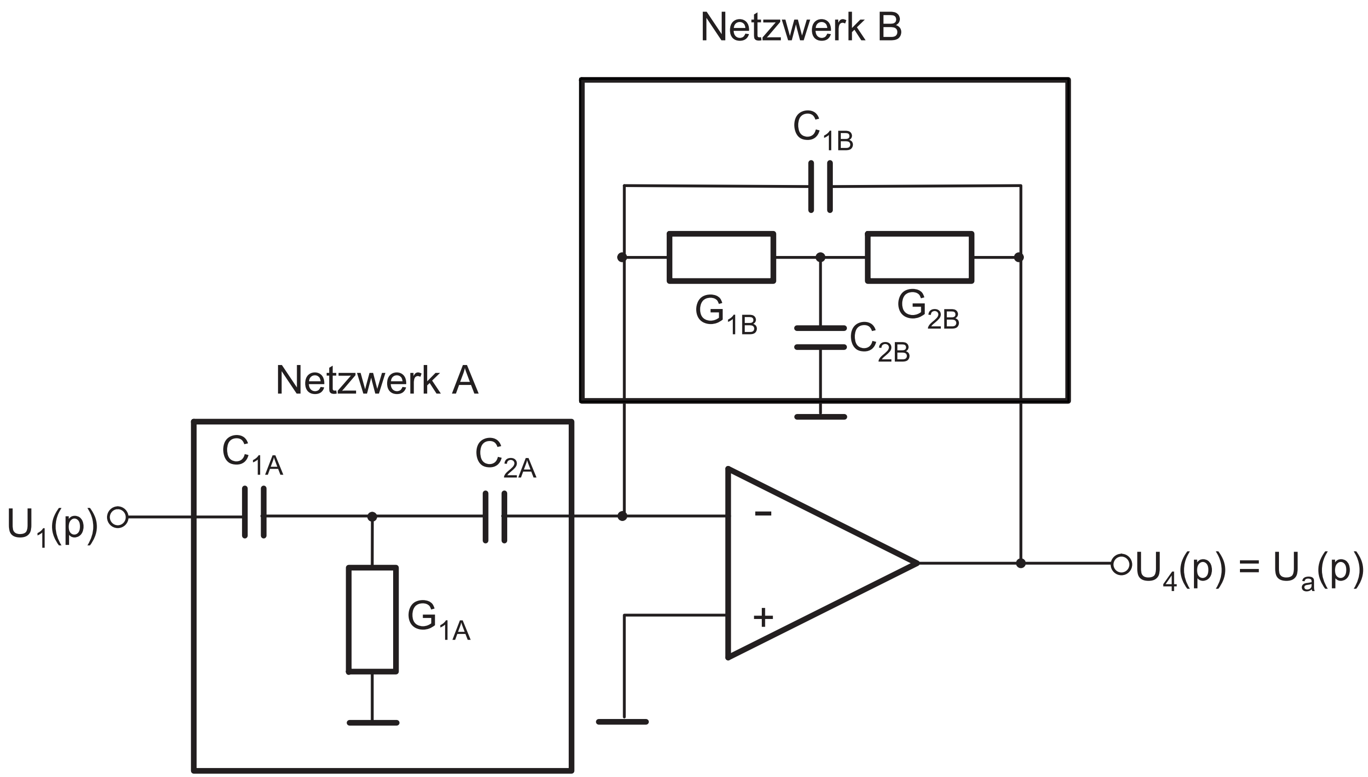
The constant H_0 gives the negative amplification for highest frequencies.

The constant b determines the behaviour around the cut-off frequency according to:

$$|H_L(j\omega_g)| = \frac{H_0}{b}$$

The realization of the system function using an ideal op-amp is illustrated in the following figure.

3.3.3 The active 2nd order RC high-pass



Active 2nd order RC high-pass with an operational amplifier

3.3.3 The active 2nd order RC high-pass

Just like the active RC low-pass for the system function of the high-pass shown in the figure also applies :

$$H_L(p) = -\frac{Y_{21A}(p)}{Y_{12B}(p)}$$

The admittance function of network B is the same as for the second order RC low-pass, so it holds (see S.21):

$$Y_{12B}(p) = -C_{1B} \cdot \frac{p^2 + \frac{G_{1B}+G_{2B}}{C_{2B}} \cdot p + \frac{G_{1B} \cdot G_{2B}}{C_{1B} \cdot C_{2B}}}{p + \frac{G_{1B}+G_{2B}}{C_{2B}}}$$

For the high-pass and network A it follows (with respect to the T-circuit of S. 19 and 20):

$$Z_1 = \frac{1}{pC_{1A}} \qquad Z_2 = \frac{1}{pC_{2A}} \qquad Z_0 = \frac{1}{G_{1A}}$$

3.3.3 The active 2nd order RC high-pass

$$Y_{21A}(p) = \frac{-Z_0}{Z_1 Z_2 + Z_1 Z_0 + Z_0 Z_2}$$

$$= -\frac{1}{G_{1A}\left(\dfrac{1}{p^2 C_{1A} C_{2A}} + \dfrac{1}{p C_{1A} G_{1A}} + \dfrac{1}{p C_{2A} G_{1A}}\right)} = -\frac{p^2 C_{1A} C_{2A}}{G_{1A}\left(1 + p\dfrac{C_{2A}}{G_{1A}} + p\dfrac{C_{1A}}{G_{1A}}\right)}$$

$$= -\frac{p^2 C_{1A} C_{2A}}{G_{1A} + p(C_{1A} + C_{2A})}$$

$$= -\frac{\dfrac{C_{1A} C_{2A}}{C_{1A} + C_{2A}} p^2}{p + \dfrac{G_{1A}}{C_{1A} + C_{2A}}}$$

3.3.3 The active 2nd order RC high-pass

Therefore one obtains for the system function of the active RC high-pass:

$$H_L(p) = -\frac{Y_{21A}(p)}{Y_{12B}(p)} = -\frac{\frac{C_{1A}C_{2A}}{C_{1B}\cdot(C_{1A}+C_{2A})}\cdot p^2 \cdot \left(p+\frac{G_{1B}+G_{2B}}{C_{2B}}\right)}{\left(p^2+\frac{G_{1B}+G_{2B}}{C_{2B}}\cdot p+\frac{G_{1B}\cdot G_{2B}}{C_{1B}\cdot C_{2B}}\right)\cdot\left(p+\frac{G_{1A}}{C_{1A}+C_{2A}}\right)}$$

Similarly as for the low-pass design the simple zero arising in this system function is waived under the following condition:

$$\frac{G_{1B}+G_{2B}}{C_{2B}} = \frac{G_{1A}}{C_{1A}+C_{2A}}$$

3.3.3 The active 2nd order RC high-pass

Under this condition, the system function becomes:

$$H_L(p) = -\frac{\dfrac{C_{1A}C_{2A}}{C_{1B}\cdot\left(C_{1A}+C_{2A}\right)}\cdot p^2}{p^2+\dfrac{G_{1B}+G_{2B}}{C_{2B}}\cdot p+\dfrac{G_{1B}\cdot G_{2B}}{C_{1B}\cdot C_{2B}}}$$

A comparison of coefficients gives for *d = 1*:

$$\frac{C_{1A}C_{2A}}{C_{1B}\cdot\left(C_{1A}+C_{2A}\right)} = H_0 \qquad \frac{G_{1B}+G_{2B}}{C_{2B}} = b\cdot\omega_g$$

$$\frac{G_{1B}\cdot G_{2B}}{C_{1B}\cdot C_{2B}} = \omega_g^2$$

3.3.3 The active 2nd order RC high-pass

One can set for example three further conditional equations:

$$C_{1A} = C_{2A} \qquad G_{1B} = \frac{1}{R_{1B}} = k \cdot G_W \qquad C_{1B} = \frac{1}{\omega_g \cdot R_W} = \frac{G_W}{\omega_g}$$

With these 3 additional relations the previous 4 relations can be rewritten like this:

$$(1) \quad \frac{C_{1A} \cdot C_{2A}}{C_{1B}(C_{1A} + C_{2A})} = H_0 \Rightarrow \quad \omega_g \frac{C_{1A}}{2G_w} = H_0 \quad (1a)$$

$$(2) \quad \frac{G_{1B} + G_{2B}}{C_{2B}} = b\omega_g \quad \Rightarrow \quad \frac{kG_w + G_{2B}}{C_{2B}} = b\omega_g \quad (2a)$$

$$(3) \quad \frac{G_{1B} G_{2B}}{C_{1B} C_{2B}} = \omega_g^2 \quad \Rightarrow \quad \frac{kG_W G_{2B}}{G_W C_{2B}} = \omega_g = \frac{kG_{2B}}{C_{2B}} \quad (3a)$$

$$(4) \quad \frac{G_{1B} + G_{2B}}{C_{2B}} = \frac{G_{1A}}{C_{1A} + C_{2A}} \quad \Rightarrow \quad \frac{kG_w + G_{2B}}{C_{2B}} = \frac{G_{1A}}{2C_{1A}} \quad (4a)$$

3.3.3 The active 2nd order RC high-pass

(1a) $\Rightarrow \quad C_{1A} = \dfrac{H_0 2G_w}{\omega_g}$

(2a) inserted in (4a) gives:

$$b\omega_g = \frac{G_{1A}}{2C_{1A}} \quad \Rightarrow \quad G_{1A} = 2b\omega_g C_{1A} = 2b\omega_g \frac{H_0 2G_w}{\omega_g} = 4bH_0 G_w$$

(2) and (3) combined gives:

$$G_{1B} + G_{2B} = C_{2B} b\omega_g \quad \Rightarrow \quad C_{2B} = \frac{G_{1B} + G_{2B}}{b\omega_g}$$

$$G_{1B} \cdot G_{2B} = C_{1B} C_{2B} \omega_g^2$$

3.3.3 The active 2nd order RC high-pass

Inserting C_{2B} in the last equation gives:

$$G_{1B}G_{2B} = C_{1B}\frac{(G_{1B}+G_{2B})}{b\omega_g}\omega_g^2 = \frac{C_{1B}G_{1B}+C_{1B}G_{2B}}{b}\omega_g$$

$$\Rightarrow G_{2B}\left(G_{1B}-\frac{C_{1B}\omega_g}{b}\right) = \frac{C_{1B}G_{1B}\omega_g}{b}$$

$$\Rightarrow G_{2B} = \frac{C_{1B}G_{1B}\omega_g}{b\left(G_{1B}-C_{1B}\frac{\omega_g}{b}\right)} = \left.\frac{C_{1B}G_{1B}\omega_g}{bG_{1B}-C_{1B}\omega_g}\right|_{G_{1B}=kG_w \text{ and } C_{1B}=\frac{G_w}{\omega_g}}$$

$$= \frac{\frac{G_w}{\omega_g}G_w k\omega_g}{bG_w k-\frac{G_w}{\omega_g}\omega_g} = \frac{G_w k}{bk-1}$$

with $bk > 1$ to ensure positive admittance

3.3.4 The active RC band-pass 2. Order

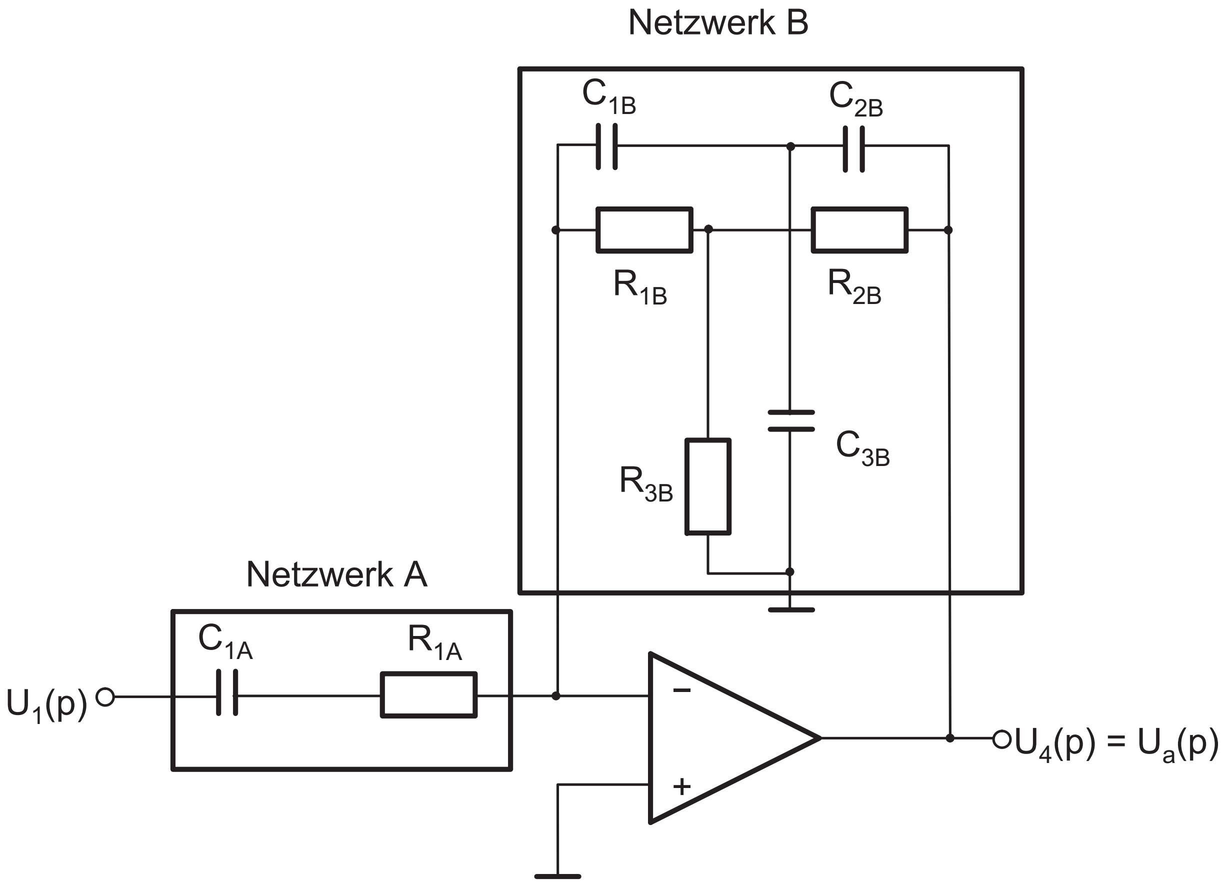
The transmission characteristics of the general stable second order band-pass is described by the system function

$$H_L(p) = \frac{H_0 \cdot \omega_0 \cdot p}{p^2 + b \cdot \omega_0 \cdot p + \omega_0^2} \quad \text{with } \omega_g > 0, H_0 > 0, \text{ real and } 0 < b < 2$$

with $H_L(j\omega_0) = \frac{H_0}{b}$

The realization of the system function according to the “Method of active RC two-port with ideal Op-amp and individual feedback path“ is illustrated in the following figure.

3.3.4 The active RC band-pass 2. Order
Version 1



Active 2nd order RC band-pass filter with an operational amplifier

3.3.4 The active RC band-pass 2. Order

Network B now is separated into 2 T-circuits. For any T-circuit (see S. 19) it holds:

$$Y_{12T} = \frac{-Z_0}{\Delta Z}$$

Thus for the Resistor T-circuit with $Z_0 = R_{3B} \quad Z_{1,2} = R_{1,2B}$ it holds:

$$Y_{12TR} = \frac{-R_{3B}}{R_{1B}R_{2B} + R_{1B}R_{3B} + R_{3B}R_{2B}} = \frac{-1/G_{3B}}{\dfrac{1}{G_{1B}G_{2B}} + \dfrac{1}{G_{1B}G_{3B}} + \dfrac{1}{G_{2B}G_{3B}}}$$

$$= \frac{-1}{\dfrac{G_{3B}}{G_{1B}G_{2B}} + \dfrac{1}{G_{1B}} + \dfrac{1}{G_{2B}}} = \frac{-G_{1B}G_{2B}}{G_{3B} + G_{2B} + G_{1B}} = \frac{-G_{1B}G_{2B}}{\sum G}$$

3.3.4 The active RC band-pass 2. Order

Likewise for the capacitor-T-circuit it holds:

$$Y_{12TC} = -\frac{p^2 C_{1B} C_{2B}}{p(C_{3B} + C_{2B} + C_{1B})}$$

$$= -\frac{p C_{1B} C_{2B}}{C_{3B} + C_{2B} + C_{1B}} = -\frac{p C_{1B} C_{2B}}{\sum C}$$

So finally for network B it holds:

$$\Rightarrow \quad Y_{12B} = Y_{12TR} + Y_{12TC} = -\frac{G_{1B} G_{2B}}{\sum G} - \frac{p C_{1B} C_{2B}}{\sum C}$$

$$= -\frac{G_{1B} G_{2B} \sum C + p C_{1B} C_{2B} \sum G}{\sum G \cdot \sum C} = -\frac{p + \dfrac{G_{1B} G_{2B} \sum C}{C_{1B} C_{2B} \sum G}}{\dfrac{\sum C}{C_{1B} C_{2B}}}$$

3.3.4 The active RC band-pass 2. Order

Analysis of network A gives (see S. 21):

$$Y_{21A} = Y_{12A} = -Y_0 = -\frac{1}{R_{1A} + \dfrac{1}{pC_{1A}}} = -\frac{pC_{1A}}{pR_{1A}C_{1A} + 1} = -\frac{G_{1A}p}{p + G_{1A}/C_{1A}}$$

So finally we arrive with the resulting system function:

$$H_L(p) = -\frac{Y_{21A}}{Y_{12B}} = \frac{G_{1A}p}{p + G_{1A}/C_{1A}} \cdot \frac{\dfrac{\Sigma C}{C_{1B}C_{2B}}}{p + \dfrac{G_{1B}G_{2B}\sum C}{C_{1B}C_{2B}\sum G}}$$

$$= -\frac{G_{1A}\Sigma C}{C_{1B}C_{2B}} \cdot \frac{p}{p^2 + p\left(\dfrac{G_{1A}}{C_{1A}} + \dfrac{G_{1B}G_{2B}\sum C}{C_{1B}C_{2B}\sum G}\right) + \dfrac{G_{1A}G_{1B}C_{2B}\sum C}{C_{1A}C_{1B}C_{2B}\sum G}}$$

3.3.4 The active RC band-pass 2. Order

A comparison with the desired system function

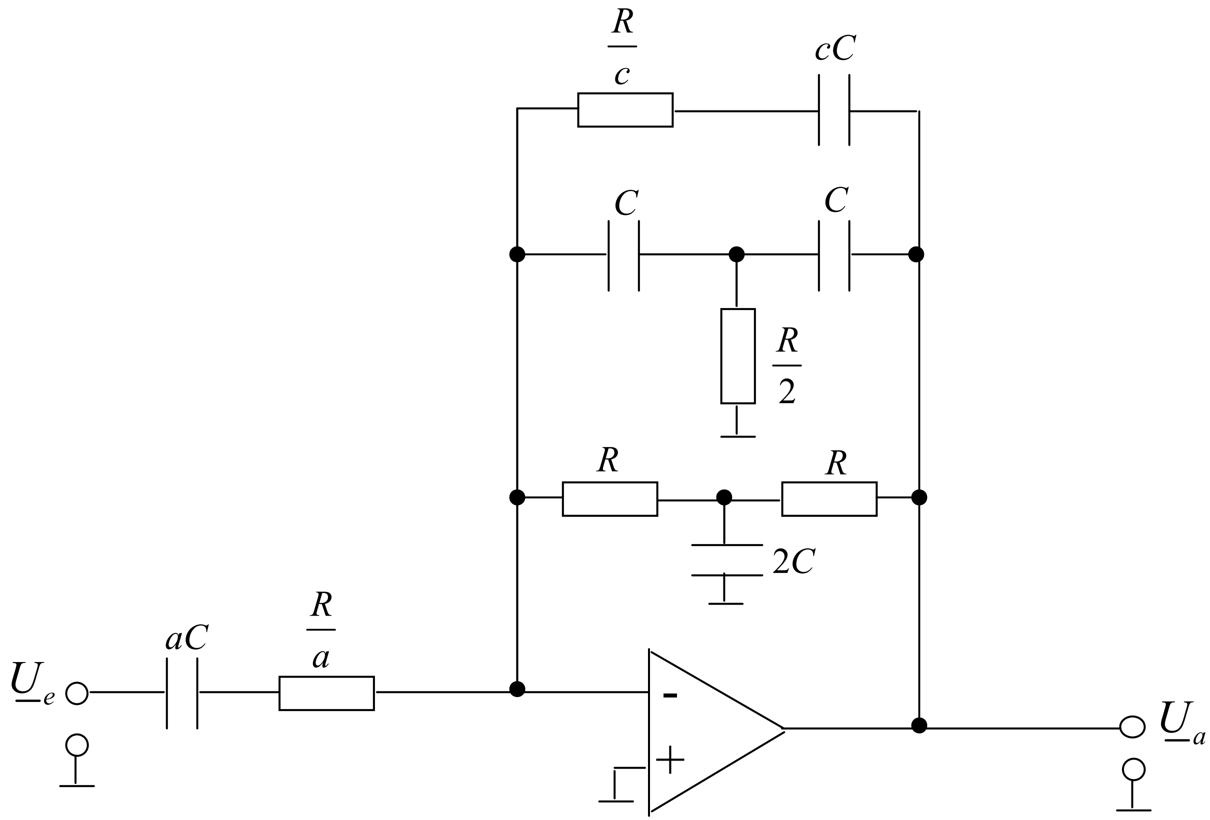
$$H_L(p) = \frac{H_0 \cdot \omega_0 \cdot p}{p^2 + b \cdot \omega_0 \cdot p + \omega_0^2}$$

gives the following relations:

$$H_0\omega_0 = \frac{G_{1A}\Sigma C}{C_{1B}C_{2B}} \qquad b\omega_0 = \left(\frac{G_{1A}}{C_{1A}} + \frac{G_{1B}G_{2B}\sum C}{C_{1B}C_{2B}\sum G} \right) \qquad \omega_0^2 = \frac{G_{1A}G_{1B}C_{2B}\sum C}{C_{1A}C_{1B}C_{2B}\sum G}$$

Another network with simplified calculation is shown in the next slides (according to “Tietze/Schenk, Halbleiter-Schaltungstechnik”):

3.3.4 The active RC band-pass 2. Order
Version 2



Band-pass filter with simple feedback

3.3.4 The active RC band-pass 2. Order

Here we see the following relations (due to slides 43 and 22):

$$Y_{21A} = -\frac{1}{Z_0} = -\frac{1}{R/a + 1/(paC)} = -\frac{paC}{1 + paC\frac{R}{a}} = -\frac{paC}{1 + pRC}$$

$$Y_{12T} = \frac{-Z_0}{Z_1 Z_2 + Z_0(Z_1 + Z_2)}$$ is then evaluated for next 2 subcircuits:

$$Y_{12B\,Part1} = -\frac{\frac{1}{2pC}}{R^2 + 2(\frac{R}{2pC})} = -\frac{1}{2pR^2C + 2R} = -\frac{1}{2R(1 + pRC)}$$ (Low-pass)

$$Y_{12B\,Part2} = -\frac{R/2}{\frac{1}{p^2C^2} + \frac{R}{2}\frac{1}{pC}} = -\frac{p^2C^2R}{2(1 + pRC)}$$ (High-pass)

$$Y_{12B\,Part3} = -\frac{1}{Z_0} = -\frac{1}{\frac{R}{c} + \frac{1}{pcC}} = -\frac{pcC}{1 + \frac{R}{c}pcC} = -\frac{pcC}{1 + pRC}$$ (according to Y_{21A} result)

3.3.4 The active RC band-pass 2. Order

$$\Rightarrow Y_{12B} = Y_{12Part1} + Y_{12Part2} + Y_{12Part3}$$

$$= -\frac{1}{2R(1+pRC)} - \frac{p^2C^2R}{2(1+pRC)} - \frac{pcC}{1+pRC}$$

$$= -\frac{1}{(1+pRC)}\left(\frac{1}{2R} + \frac{p^2RC^2}{2} + pcC\right)$$

$$= -\frac{1}{2R} \cdot \frac{1}{(1+pRC)} \cdot (1 + p^2R^2C^2 + 2pcRC)$$

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3.3.4 The active RC band-pass 2. Order

$$H_L(p) = -\frac{Y_{21A}}{Y_{12B}} = -\frac{paC}{1+pRC}\frac{2R(1+pRC)}{1+2pcRC+p^2R^2C^2}$$

$$= -\frac{paC\cdot 2R}{1+2pcRC+p^2R^2C^2}$$

$$= -\frac{\frac{2a}{RC}p}{p^2+p\frac{2c}{RC}+\frac{1}{R^2C^2}}$$

$$\overset{!}{=} -\frac{H_0\omega_0 p}{p^2+b\omega_0 p+\omega_0^2} \qquad \Rightarrow \qquad \omega_0 = \frac{1}{RC}$$

$$b\omega_0 = \frac{2c}{RC} \Rightarrow c = \frac{\omega_0 bRC}{2} = \frac{b}{2}$$

$$H_0\omega_0 = \frac{2a}{RC} \Rightarrow a = H_0/2$$

There are 4 unknowns (*R*, *C*, *a*, *c*) and there are 3 conditional relations.

ω_0, H_0 and *b* must be given.

So typically C is set to a convenient value to be able to solve the system of 3 equations.