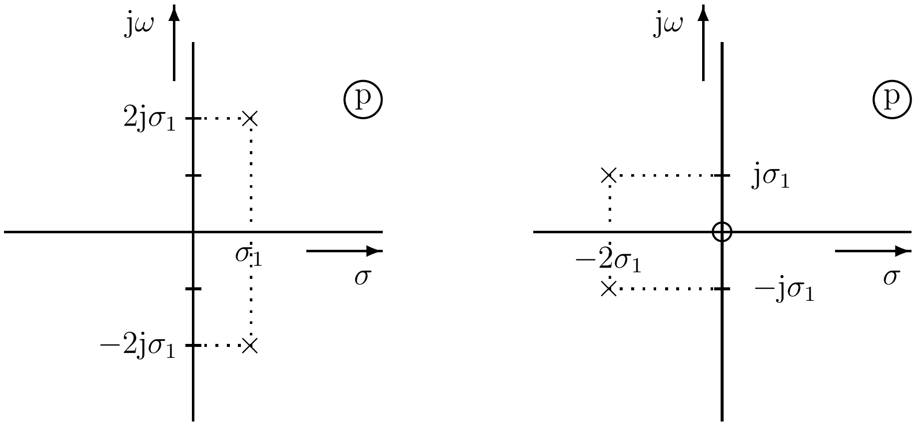
Chapter 1 – Introduction and Basics

Problem 1.1

The following two pole-zero-plots describe the system functions of the two-ports A and B, whereby the value $\sigma_1 > 0$.



1.1.1 Determine the system functions $H_{\mathrm{LA}}(p)$ and $H_{\mathrm{LB}}(p)$ of both two-ports.

1.1.2 Determine the transfer functions $H_{\mathrm{A}}(\omega) = H_{\mathrm{LA}}(p = \mathrm{j}\omega)$ and $H_{\mathrm{B}}(\omega) = H_{\mathrm{LB}}(p = \mathrm{j}\omega)$ of both two-ports.

1.1.3 Which kind of systems are described by the two-ports A and B. Justify your answer.

1.1.4 Determine the impulse response $h_{\mathrm{A}}(t)$ and $h_{\mathrm{B}}(t)$ of the system functions $H_{\mathrm{LA}}(p)$ and $H_{\mathrm{LB}}(p)$.

1.1.5 Give a short comment on the stability of both two-ports.

Problem 1.2

The following system function $H_{\mathrm{L}}(p)$ of a linear, time invariant two-port is given:

$$H_{\mathrm{L}}(p) = \frac{U_2(p)}{U_1(p)} = \frac{p}{p^2 + 2ap + b} \qquad \text{with } a, b \text{ real and positive}$$

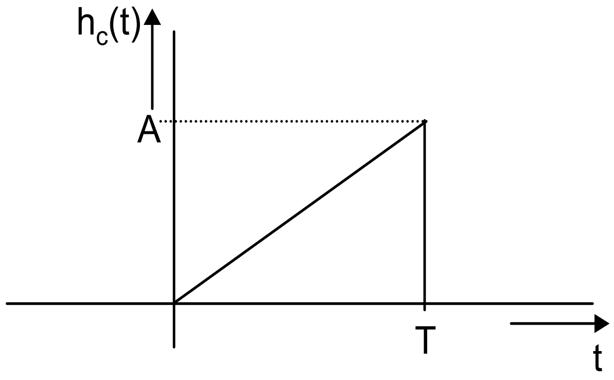
1.2.1 Determine the poles and zeros and name the conditions for $H_{\mathrm{L}}(p)$ being a system function of a stable two-port.

1.2.2 Calculate the impulse response $h(t)$, which is the back-transformed system function $H_{\mathrm{L}}(p)$, for $a = 3$ and $b = 8$.

1.2.3 Which kind of system is described by the two-port?

Problem 1.3

The following impulse response $h_\mathrm{c}(t)$ should be approximated using a linear, time invariant two-port.



1.3.1 Describe $h_\mathrm{c}(t)$ as a sum of weighted and time-shifted ramp- and step-functions.

1.3.2 Determine the Laplace-Transformed $H_\mathrm{Lc}(p)$ of $h_\mathrm{c}(t)$ using the result of 1.3.1.

1.3.3 Using $H_\mathrm{Lc}(p)$ develop an approximated function $H_\mathrm{La}(p)$ by substituting the term e^{pT} by $\mathrm{e}^{pT} = 1 + pT + (pT)^2/2 + \ldots$, truncating after the third term of the series expansion.

1.3.4 Draw the pole-zero-plot of $H_\mathrm{La}(p)$.

1.3.5 Determine $h_\mathrm{a}(t)$, which is the Laplace-back-transformed of $H_\mathrm{La}(p)$ and sketch $h_\mathrm{a}(t)$.

Problem 1.4

Given is a graph with n nodes, in which each node is connected to all other nodes by a branch.

1.4.1 Draw the graphs for $n = 2(1)5$.

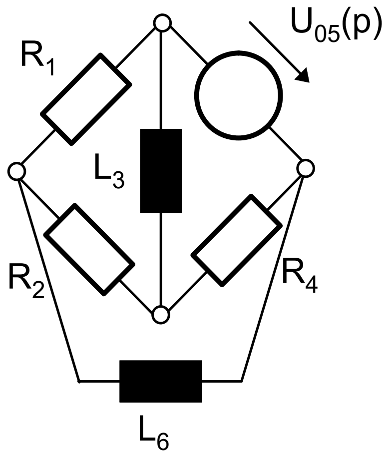
1.4.2 Derive a formula for the calculation of the number $z(n)$ of branches of each graph.

1.4.3 Draw for $n = 2(1)5$ all complete trees of each graph.

1.4.4 Derive the formula for the calculation of the number $z_\mathrm{u}(n)$ of independent branches of a graph.

Problem 1.5

The following network is given:



1.5.1 Outline a complete tree.

1.5.2 Determine the incidence matrix $\overleftrightarrow{M}$ of the previously outlined tree.

1.5.3 Determine the branch impedance matrix $\overleftrightarrow{Z}_{\text{BI}}$.

1.5.4 Determine the loop impedance matrix $\overleftrightarrow{Z}$.

<u>Problem 1.6</u>

Given is the following incidence matrix:

$$\overleftrightarrow{M}= \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

which connects the branch current vector $\vec{I}$ with the loop current vector $\vec{J}$ according to $\vec{I} = \overleftrightarrow{M} \cdot \vec{J}$. Further on the branch impedance matrix

$$\overleftrightarrow{Z}_{\mathrm{BI}}= \begin{pmatrix} 1/pC_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & pL_3 & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 & 1/pC_5 \end{pmatrix}$$

of the network is given. This network is powered by an independent voltage source with the voltage $U_{\mathrm{s}\nu}(p)$.

1.6.1 Determine the number k of nodes.

1.6.2 Denote the loop currents $\vec{J}_\mu(p)$ and the independent branches of the network by specifying the indices μ.

1.6.3 Determine the loop impedance matrix $\overleftrightarrow{Z}$ of the network.

1.6.4 Determine the voltage source vector $\vec{U}_{\mathrm{s}}(p)$.

1.6.5 Draw the corresponding directed graph of the network, insert the branch indices ν and indicate the complete tree of the graph by drawing the corresponding branches with thick lines.

1.6.6 Draw the network using the symbols of the network elements and denote those elements.

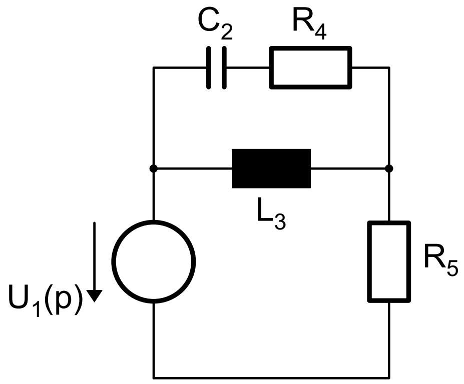
1.6.7 Determine the components $J_\mu(p)$ of the loop current vector $\vec{J}(p)$.

1.6.8 Determine the components $I_\nu(p)$ of the branch current vector $\vec{I}(p)$.

1.6.9 Determine the components $U_\nu(p)$ of the branch voltage vector $\vec{U}(p)$.

Problem 1.7

The following network is given.



1.7.1 Draw a directed graph of the network and underline the branches of one complete tree. Denote each branch and the loop currents $J_\mu(p)$.

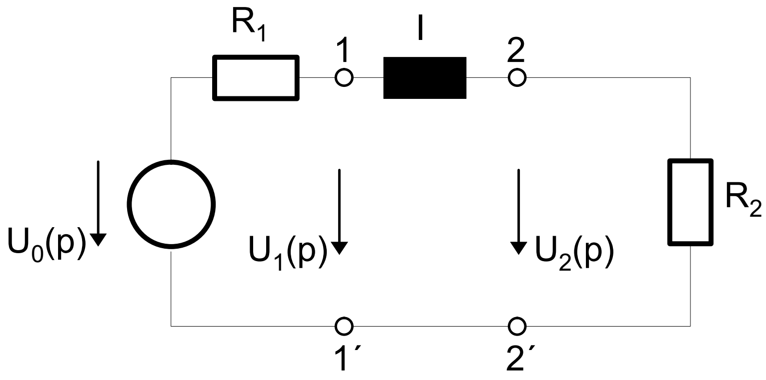
1.7.2 Calculate the number z_{u} of independent branches.

1.7.3 Determine the incidence matrix $\overset{\leftrightarrow}{M}$, which links the loop current vector $\vec{J}$ and the branch current vector $\vec{I}$ according to $\vec{I}(p) = \overset{\leftrightarrow}{M} \cdot \vec{J}(p)$

1.7.4 Determine the branch impedance matrix $\overset{\leftrightarrow}{Z}_{\mathrm{BI}}$, the loop impedance matrix $\overset{\leftrightarrow}{Z}$ and the branch voltage vector $\vec{U}(p)$ of the network.

Problem 1.8

The following network is given.



1.8.1 Determine the cascade matrix $\overleftrightarrow{A}$ of the two-port, which is given by the nodes 1 and 1' at the input and 2 and 2' at the output.

1.8.2 Determine the 'operational effective function' $H_{\mathrm{LB}}(p)$.

1.8.3 Plot the pole-zero-diagram of $H_{\mathrm{LB}}(p)$.

1.8.4 Determine the normalised function $H_{\mathrm{LBn}}(\rho)$ with

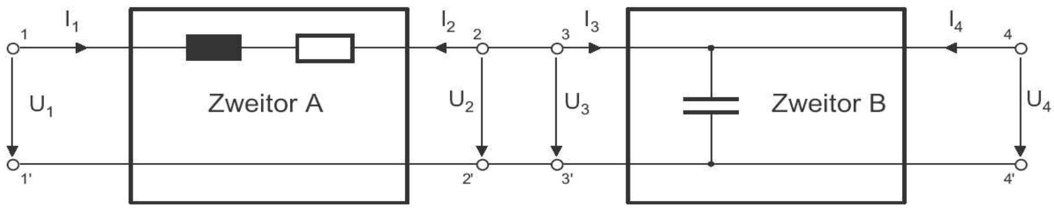
$$\rho = \frac{p}{\omega_{\mathrm{n}}} = \frac{\sigma}{\omega_{\mathrm{n}}} + \mathrm{j}\frac{\omega}{\omega_{\mathrm{n}}} = \varsigma + \mathrm{j}\Omega$$

using the normalised angular frequency ω_{n}, which is the -3 dB-cut-off-frequency of $|H_{\mathrm{LB}}(\mathrm{j}\omega)|$. Further on use the normalising resistance R_{N} which corresponds to the resistance R_1.

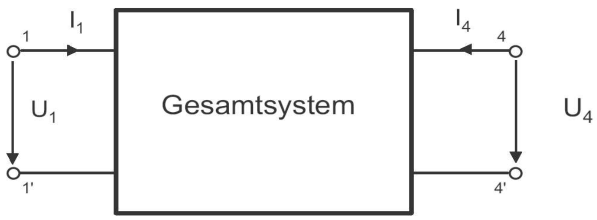
1.8.5 Determine the normalised transfer function $H_{\mathrm{Fn}}(\Omega) = H_{\mathrm{LBn}}(\mathrm{j}\Omega)$.

Problem 1.9

Given is the following network



and its equivalent circuit diagram

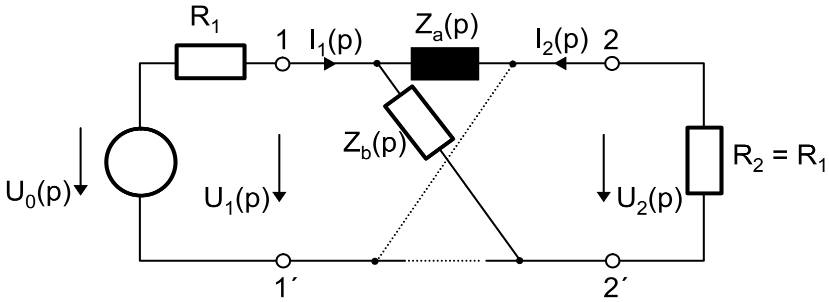


1.9.1 Determine the cascade matrices $\overleftrightarrow{A}_{\mathrm{a}}$ and $\overleftrightarrow{A}_{\mathrm{b}}$ of both two-ports, which are given by the nodes 1, 1', 2, 2', 3, 3', 4 and 4'.

1.9.2 Determine the cascade matrix $\overleftrightarrow{A}_{\mathrm{ges}}$ of the equivalent circuit diagram, where $\overleftrightarrow{A}_{\mathrm{a}}$ and $\overleftrightarrow{A}_{\mathrm{b}}$ are cascaded.

Problem 1.10

Given is the following symmetric bridge two-port



Between the impedances $Z_\mathrm{a}(p)$ and $Z_\mathrm{b}(p)$ the relation $Z_\mathrm{a} \cdot Z_\mathrm{b} = R_1^2$ is assumed.

1.10.1 Determine the impedance matrix $\overleftrightarrow{Z}$ of the two-port.

1.10.2 Determine the 'operational effective function' $H_\mathrm{LB}(p)$ of the network.

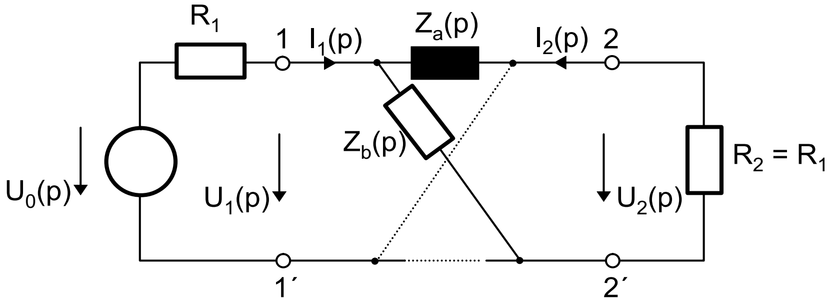
1.10.3 Determine $Z_\mathrm{a}(p)$ and $Z_\mathrm{b}(p)$ as functions of R_1 and $H_\mathrm{LB}(p)$.

1.10.4 Which condition $|H_\mathrm{LB}(p = \mathrm{j}\omega)|$ has to fulfill in order to be realised by passive RLC-elements?

Note 1: Which values $\Re\{Z_\mathrm{a}(p)\}$ and $\Re\{Z_\mathrm{b}(p)\}$ can assume?
Note 2: Substitute $H_\mathrm{LB} = H_\mathrm{r}(p) + \mathrm{j}H_\mathrm{i}(p)$.

Problem 1.11

Given is a symmetric bridge two-port



Further on $Z_\mathrm{a}(p) = pL$ and the 'operational effective function' $H_\mathrm{LB}(p)$ is given as:

$$H_\mathrm{LB}(p) = \frac{k}{(p - p_\infty)(p - p_\infty^*)}$$

with $p_\infty = -2\omega_0 + \mathrm{j}\omega_0$, $\omega_0 > 0$ and k real and positive.

1.11.1 Sketch the pole-zero-plot of $H_\mathrm{LB}(p)$ and determine the unit of k.

1.11.2 Which type of filter is described by the given $H_\mathrm{LB}(p)$? Why?

1.11.3 Which condition k has to fulfill if the given 'operational effective function' $H_\mathrm{LB}(p)$ has to be realised by passive RLC components?

1.11.4 Determine $Z_\mathrm{b}(p)$ using the result of 1.11.3

Problem 1.12

Given is a linear time invariant system with the impulse response

$$h(t) = \frac{1}{T_0}\mathrm{rect}\left(\frac{t-T}{T}\right)$$

1.12.1 Sketch the impulse response $h(t)$ as a function of the time t and chart important abscissa and ordinate values.

1.12.2 Determine the system function $H_\mathrm{L}(p) = \mathcal{L}\{h(t)\}$.

1.12.3 Determine the poles and zeros of $H_\mathrm{L}(p)$ and sketch them in a pole-zero-plot.

1.12.4 Determine the normalised impulse response $h_\mathrm{n}(\tau)$ for the normalised time $T_\mathrm{N} = 1$ ms. Further on $T_0 = 1$ ms and $T = 2$ ms. Sketch $h_\mathrm{n}(\tau)$ as function of τ.

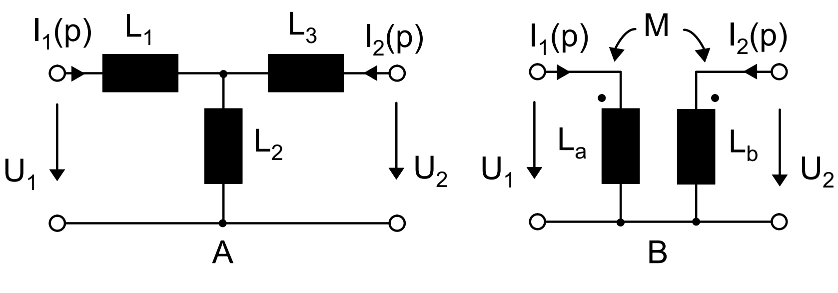
1.12.5 Determine the normalised Laplace-transformed $S_\mathrm{nL}(\rho)$ for the normalised frequency $\omega_\mathrm{N} = 1/T_\mathrm{N}$.

Problem 1.13

As a first result for the realisation of a network function, the two-port A has been calculated. This two-port is constituted by three not coupled inductances L_1, L_2 and L_3. The negative inductance L_1 is given as

$$L_1 = -|L_2| \quad \text{with} \quad |L_1| < |L_2|$$

The two-port A should be replaced by the ideal transformer B. There the primary inductance L_a is coupled to the secondary inductance L_b by $M^2 = L_\mathrm{a} \cdot L_\mathrm{b}$.



1.13.1 Determine the impedance matrices $\overleftrightarrow{Z}_\mathrm{A}$ and $\overleftrightarrow{Z}_\mathrm{B}$ of the two-ports.

1.13.2 Determine the dependence $L_3 = f(L_1, L_2)$ which is necessary for the equivalence of the two-ports.

1.13.3 With the result of 1.13.2 determine M, L_a and L_b as functions of L_1 and L_2, in order to replace the two-port A by the two-port B.

Problem 1.14

The impedances Z_a and Z_b of a symmetric bridge two-port (as shown in figure 1.20 of the manuscript) are given by

$$Z_\mathrm{a} = \frac{R_1}{pR_1C_1 + 1} \quad \text{and} \quad Z_\mathrm{b} = \frac{pR_2C_2 + 1}{pC_2} \quad \text{with } R_2 > R_1$$

1.14.1 Draw the realisations of Z_a and Z_b using passive RLC-elements and denote the components.

1.14.2 Draw the symmetric bridge two-port.

1.14.3 Using the BARTLETT' symmetry proposition, derive from the symmetric bridge two-port of 1.14.2 an equivalent two-port with a symmetrical structure without crossing branches.

Chapter 2
Characteristics and realization of passive two-terminal network function

Problem 2.1

Given is the normalised impedance function

$$Z_{\mathrm{n}}(\rho) = \frac{3\rho^4 + 8\rho^2 + 1}{\rho^3 + \rho} \quad \text{with} \quad \rho = \varsigma + \mathrm{j}\Omega$$

2.1.1 Determine the poles and zeros of $Z_{\mathrm{n}}(\rho)$ and sketch them in a pole-zero-diagram.

2.1.2 Is it possible to implement the impedance function $Z_{\mathrm{n}}(\rho)$ (after de-normalisation) using passive elements? Why or why not?

2.1.3 Expand $Z_{\mathrm{n}}(\rho)$ into a series of partial fractions and draw the corresponding circuit, naming the components.

Problem 2.2

The description of a fully normalised impedance function $Z_{\mathrm{n}}(\rho)$ of a passive two-pole as a partial fraction is

$$Z_{\mathrm{n}}(\rho) = \frac{a_0}{\rho} + \sum_{\nu=1}^{N} \frac{2a_\nu \rho}{\rho^2 + \Omega_\nu^2} + a_\infty \rho$$

with $a_0,\ a_\nu,\ a_\infty$ real and positive.

2.2.1 Show that $Z_{\mathrm{n}}(\rho = \mathrm{j}\Omega)$ is an imaginary function in Ω and that the gradient of $\mathrm{Im}\{Z_{\mathrm{n}}(\mathrm{j}\Omega)\}$ is always positive, i.e.

$$\frac{\partial}{\partial\Omega}\mathrm{Im}\{Z_{\mathrm{n}}(\mathrm{j}\Omega)\} > 0 \ \ \forall \quad \Omega$$

Problem 2.3

Given is the following normalised impedance function of a two-pole

$$Z_{\mathrm{n}}(\rho) = \frac{8\rho^4 + 80\rho^2 + 72}{\rho^3 + 4\rho}$$

2.3.1 Determine the poles and zeros of $Z_{\mathrm{n}}(\rho)$ and draw the corresponding pole-zero-plot.

2.3.2 Which kind of two-pole is described by $Z_{\mathrm{n}}(\rho)$?

2.3.3 Expand $Z_{\mathrm{n}}(\rho)$ into a sum of partial fractions and draw the corresponding circuit, naming the components.

2.3.4 Denormalise $Z_{\mathrm{n}}(\rho)$ using

$$R_{\mathrm{N}} = 100\ \Omega \qquad \text{and} \qquad \omega_{\mathrm{N}} = 10^4\ \mathrm{s}^{-1}$$

and draw the corresponding network.

Problem 2.4

Analyse if the given normalised two-pole impedance functions describe passive LC-Transformer two-poles and justify your answer.

$$Z_{\mathrm{n1}}(\rho) = \frac{\rho^2 + 3\rho + 1}{\rho}$$

$$Z_{\mathrm{n2}}(\rho) = \frac{\rho^3}{(\rho^2 + 2)^2}$$

$$Z_{\mathrm{n3}}(\rho) = \frac{(\rho^2 + 2)(\rho^2 + 4)}{\rho^2 + 8}$$

$$Z_{\mathrm{n4}}(\rho) = \frac{\rho}{\rho^2 + 3}$$

$$Z_{\mathrm{n5}}(\rho) = \frac{2\rho^4 + 11\rho^2 + 4}{\rho^3 + 4\rho}$$

$$Z_{\mathrm{n6}}(\rho) = \frac{\rho + 2}{\rho^3 + 1}$$

Problem 2.5

Given is the following normalised impedance function.

$$Z_{\mathrm{n}}(\rho) = \frac{24\rho^3 + 192\rho}{\rho^4 + 20\rho^2 + 64} \quad \text{with} \quad \rho = \frac{p}{\omega_{\mathrm{N}}} = \varsigma + \mathrm{j}\Omega$$

2.5.1 Determine the poles and zeros of $Z_{\mathrm{n}}(\rho)$ and draw the corresponding pole-zero-plot.

2.5.2 Calculate and draw the implementation of the

- 1st Foster-Structure
- 2nd Foster-Structure
- 1st Cauer-Structure
- 2nd Cauer-Structure

Problem 2.6

Given is the following normalised impedance function.

$$Z_{\mathrm{n}}(\rho) = \frac{\rho^2 + 8\rho + 12}{\rho^2 + 4\rho}$$

2.6.1 Which type of two-pole is described by $Z_{\mathrm{n}}(\rho)$, justify your answer.

2.6.2 Calculate the corresponding continued fraction arrangement, which can be obtained by subtracting the poles at $\rho = \infty$. I.e. the 1st Cauer-Form.

2.6.3 Calculate the corresponding admittance-partial-fraction arrangement, i.e. the 2nd Foster-Form.

2.6.4 De-normalise the impedance-function $Z_{\mathrm{n}}(\rho)$ using the normalised resistance and frequency

$$R_{\mathrm{N}} = 100\ \Omega \qquad \text{and} \qquad \omega_{\mathrm{N}} = 10^4\ \mathrm{s}^{-1}$$

and draw the network found in 2.6.3 with de-normalised elements.

Problem 2.9

Given is the following normalised impedance function of a passive RC-two-pole.

$$Z_{\mathrm{n}}(\rho) = \frac{\rho^2 + 4\rho + 3}{\rho^2 + 2\rho} \quad \text{with} \quad \rho = \frac{p}{\omega_{\mathrm{N}}} = \varsigma + \mathrm{j}\Omega$$

and

$$\omega_{\mathrm{N}} = 10^3\ \mathrm{s}^{-1} \quad \text{and} \quad R_{\mathrm{N}} = 100\ \Omega$$

2.9.1 Determine the linear-factor-description of $Z_{\mathrm{n}}(\rho)$.

2.9.2 Determine the poles and zeros of $Z_{\mathrm{n}}(\rho)$ and draw the corresponding pol-zero-plot.

2.9.3 Develop the impedance-partial-fraction-description (the 1st Forster-Form) of $Z_{\mathrm{n}}(\rho)$.

2.9.4 Draw the corresponding circuit with the normalised components and their values.

2.9.5 De-normalise the component-values found in 2.9.4 and give their resistance in Ohm and the capacitance in μF.

Chapter 3
Characteristics and realisation of active RC-two-ports network function

Problem 3.1

For an active 2nd order RC-high-pass, realised with the 'method of infinite gain and single feed-back path', one can describe the system function as follows:

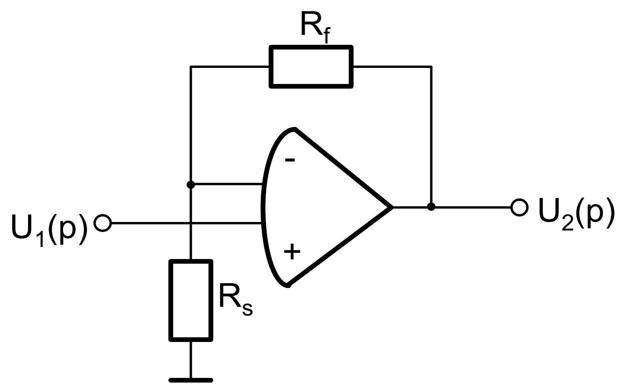
$$H_\mathrm{L}(p) = -\frac{\dfrac{C_{1A}\cdot C_{2A}}{C_{1B}\cdot(C_{1A}+C_{2A})}\cdot p^2}{p^2+\dfrac{G_{1B}+G_{2B}}{C_{2B}}\cdot p+\dfrac{G_{1B}\cdot G_{2B}}{C_{1B}\cdot C_{2B}}}$$

3.1.1 Determine the fully normalised function $H_\mathrm{Ln}(\rho)$ of $H_\mathrm{L}(p)$.

3.1.2 Determine the normalised resistances and capacitances of the active high-pass-circuit as a function of the normalised cut-off frequency $\Omega_\mathrm{g} = \omega_\mathrm{g}/\omega_\mathrm{N}$ and the constants H_0 and b.

Problem 3.2

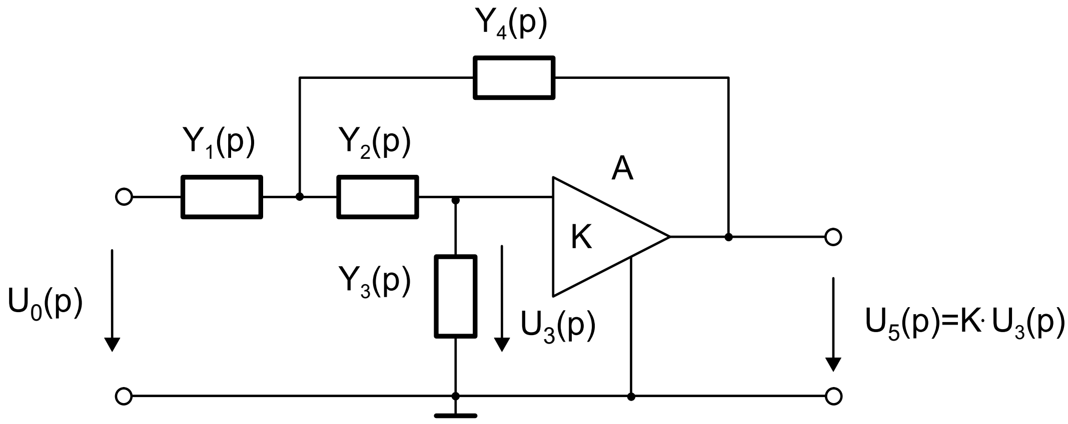
Given is the following circuit, constituted by an ideal operational amplifier and two resistances:



3.2.1 Determine the system function $H_\mathrm{L}(p) = U_2(p)/U_1(p)$.

Problem 3.3

Given is the following circuit with the active voltage-driven voltage-source A (input impedance $Z_\mathrm{e} \to \infty$):



3.3.1 Determine the system function $H_\mathrm{L}(p) = U_5(p)/U_0(p)$ as a function of $Y_1(p),\ Y_2(p),\ Y_3(p),\ Y_4(p)$ and k.

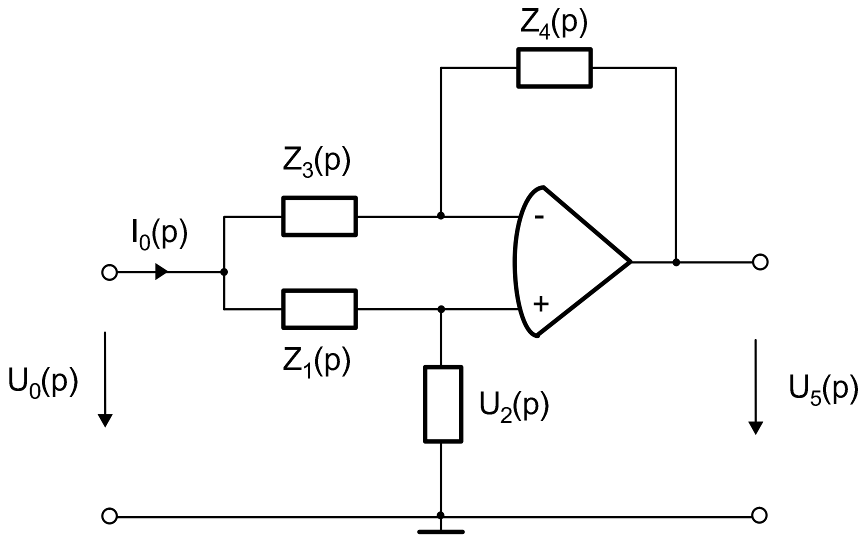
3.3.2 Determine the system function $H_\mathrm{L}(p)$ for the case that $Y_1(p)$ is the admittance of a resistor R_1, $Y_2(p)$ is the admittance of a resistor R_2, $Y_3(p)$ is the admittance of a capacitor with the capacitance C_3 and $Y_4(p)$ is the admittance of a capacitor with the capacitance C_4.

3.3.3 Which kind of filter is described by the circuit shown in 3.3.2?

3.3.4 Determine the input impedance $Z_\mathrm{e} = U_0(p)/I_1(p)$ of the circuit as a function of the admittances $Y_1(p),\ Y_2(p),\ Y_3(p),\ Y_4(p)$ and the constant k.

Problem 3.4

Given is the following circuit with the ideal operational amplifier:



3.4.1 Determine the system function $H_L(p) = U_5(p)/U_0(p)$ as a function of the impedances $Z_1(p)$, $Z_2(p)$, $Z_3(p)$ and $Z_4(p)$; and also as a function of the admittances $Y_1(p)$, $Y_2(p)$, $Y_3(p)$, and $Y_4(p)$.

3.4.2 Determine the input impedance $Z_e(p) = U_0(p)/I_0(p)$.

3.4.3 The impedances $Z_1(p)$, $Z_2(p)$, $Z_3(p)$ and $Z_4(p)$ shall be the impedances of resistors or capacitors. Draw a circuit constituted by these elements, which describes a 1st order active allpass.