Chapter 3 – Design of IIR Filter-Systems

Problem 3.1

Given is the system-function:

$$H_{\mathrm{L}}(p) = \frac{a(p - p_{01})}{(p - p_{\infty 1})(p - p_{\infty 1}^{*})}$$

with

$$a \text{ positive and real}, \quad p_{01} = -|\sigma_{01}|, \quad p_{\infty 1} = -|\sigma_{\infty 1}| + \mathrm{j}\omega_{\infty 1}$$

of an analogue system, which shall be used for the realisation of a digital system.

3.1.1 Draw the pole-zero-plot of $H_{\mathrm{L}}(p)$.

3.1.2 Determine the function $H_{\mathrm{L}}(-p)$ and draw the pole-zero-plot of $H_{\mathrm{L}}(-p)$.

3.1.3 Determine the function $A_{\mathrm{L}}(p) = H_{\mathrm{L}}(p) \cdot H_{\mathrm{L}}(-p)$ and draw the pole-zero-plot of $A_{\mathrm{L}}(p)$.

3.1.4 Calculate the approximating function $A(\omega) = A_{\mathrm{L}}(p = \mathrm{j}\omega)$.

3.1.5 Which is the relation between $A(\omega)$ and the Fourier-transformed $H_{\mathrm{F}}(\omega)$ of the impulse response $h(t)$ of the analogue system?

3.1.6 Calculate the $A(\omega)$ under the conditions:

$$|\sigma_{\infty 1}| = \omega_{\infty 1} \qquad |\sigma_{01}| = 0 \qquad a = 1$$

3.1.7 With the conditions described in 3.1.6 and the normalised frequency $\omega_{\mathrm{N}} = \omega_{\infty 1}$, determine the (frequency-) normalised approximating function $A_{\mathrm{n}}(\Omega) = A(\Omega \cdot \omega_{\mathrm{N}})$. Afterwards draw the fully normalised approximating function $A_{\mathrm{nn}}(\Omega)$ in the range $0 \le \Omega \le 10$ for $\Omega_m = m \cdot 0.5$ with $m = 0(1)20$.

3.1.8 With the conditions described in 3.1.6 calculate the fully normalised magnitude characteristic $|H_{\mathrm{Fnn}}(\Omega)|$ of the transfer-function $|H_{\mathrm{F}}(\omega)|$ of the analogue reference system.

Problem 3.2

Given is the approximating function:

$$A(\omega) = \frac{2 \cdot \left(\frac{\omega}{\omega_N}\right)^2}{1 + \left(\frac{\omega}{\omega_N}\right)^4} = |H_F(\omega)|^2 \quad \text{with} \quad \omega_N = \text{normalising frequency}$$

of an analogue system with the transfer-function $H_F(\omega)$.

3.2.1 Draw the (frequency) normalised approximating function $A_n(\Omega) = A(\Omega \cdot \omega_N)$ as a function of the angular frequency $\Omega = \omega/\omega_N$ giving important characterising values.

3.2.2 Which kind of filter is described by the analogue system?

3.2.3 Insert in the drawing of 3.2.1 the limits of the pass- and stop-band of $A_n(\Omega)$. The pass-band is defined by the tolerance δ_d, with ordinate values between 1 and $(1 - \delta_d)^2$. The stop-band is defined by the tolerance δ_s, with ordinate values lower than $(\delta_s)^2$.

For values $(1 - \delta_d)^2 = 0.9$ and $(\delta_s)^2 = 0.3$ insert the normalised frequencies $\Omega_{\pm d}$ for the pass-band and $\Omega_{\pm s}$ for the stop-band respectively.

3.2.4 Determine the normalised frequencies $\Omega_{\pm d}$ and $\Omega_{\pm s}$ as functions of the tolerances δ_d and δ_s.

3.2.5 Determine the relation between the frequencies ω_{-d} and ω_{+d} and the normalising frequency ω_N. Therefore calculate the geometric mean of the pass-frequencies Ω_{+d} and Ω_{-d}.

3.2.6 Determine the relation between the frequencies ω_{-s} and ω_{+s} and the normalising frequency ω_N. Therefore calculate the geometric mean of the stop-frequencies Ω_{+s} and Ω_{-s}.

3.2.7 Determine the system-function $H_L(p)$ of the analogue system and draw the corresponding pole-zero-plot.

3.2.8 Determine the absolute value of the frequency normalised transfer-function $|H_{Fn}(\Omega)| = |H_F(\Omega \cdot \omega_N)|$.

3.2.9 Calculate and draw the function $|H_{Fn}(\Omega_\nu)|$ for $\Omega_\nu = \nu \cdot 0.2$ in the range $\nu = 0(1)15$. In addition insert the tolerance boundaries for $(1 - \delta_d)^2 = 0.9$ and $\delta_s^2 = 0.3$ and the corresponding boundary frequencies.