Chapter 2 – Design of FIR Filter-Systems

Problem 2.1

During an infra-sound measurement a signal $s(t)$ is observed, which shows an maximal frequency of $f_g = 16$ Hz. This signal should be filtered by a digital band-pass, with an upper cut-off frequency of $f_s = 9$ Hz.

2.1.1 Determine the minimal sampling frequency $f_{ak,min}$ the signal $s(t)$ has to be sampled.

2.1.2 Determine the related maximal sampling period $T_{ak,max}$ and compare it with the sampling period T_a when sampling with the Nyquist rate.

Problem 2.2

A video signal $s(t)$ with a bandwith of $f_g = 5$ MHz has to be processed by a digital FIR low-pass, characterised by a cut-off frequency of $f_s = 200$ kHz.

2.2.1 Determine the minimal sampling frequency $f_{ak,min}$ the signal $s(t)$ has to be sampled.

2.2.2 Determine the related maximal sampling period $T_{ak,max}$.

2.2.3 How long the maximal time T_M of one 'multiply and accumulate' cycle can be, if the digital FIR low-pass is constituted by $N = 19$ filter-coefficients, having to be processed in series?

Problem 2.3

Given is the wanted transfer-function of an ideal digital low-pass:

$$H_{\mathrm{wF}}(\omega) = H_{\mathrm{bFTP}}(\omega) * \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_{\mathrm{a}})$$

For the baseband-function $H_{\mathrm{bFTP}}(\omega)$ the following holds:

$$H_{\mathrm{bFTP}}(\omega) = \begin{cases} 1 & \forall \quad 0 \leq |\omega| \leq \omega_{\mathrm{T}} \quad \text{with} \quad \omega_{\mathrm{T}} < \frac{\omega_{\mathrm{a}}}{2} \\ 0 & \forall \quad |\omega| > \omega_{\mathrm{T}} \end{cases}$$

A causal digital FIR low-pass shall be designed, whos magnitude characteristics $|H_{\mathrm{rF}}(\omega)|$ approximate the magnitude characteristics $|H_{\mathrm{wF}}(\omega)|$ of the given ideal digital low-pass.

2.3.1 Derive a general equation for the calculation of the values of the unweighted impulse sequence $h_{\mathrm{bk}}(k)$ of the causal digital FIR low-pass. Given are the low-pass cut-off-frequency ω_{T}, the sampling- (and clock-) frequency ω_{a} and the number of filter coefficients $N = 2N_{\mathrm{f}} + 1$.

2.3.2 Determine the general equation for the calculation of the values of the weighted impulse sequence $h_{\mathrm{rk}}(k) = h_{\mathrm{wk}}(k) \cdot h_{\mathrm{bk}}(k)$ of the causal digital FIR low-pass, if the coefficients of the weighting function $h_{\mathrm{wk}}(k)$ are equal to a Kaiser-sequence $h_{\mathrm{Kk}}(k)$.

2.3.3 Determine the values of the weighted impulse sequence $h_{\mathrm{rk}}(k)$ of the causal digital FIR low-pass and plot them over k. The following data for the realisation of the filter is given:

- Low-pass cut-off-frequency $f_{\mathrm{T}} = \omega_{\mathrm{T}}/2\pi = 12$ kHz
- Clock-frequency $f_{\mathrm{a}} = \omega_{\mathrm{a}}/2\pi = 48$ kHz
- Number of filter coefficients $N = 9$
- Weighting of the baseband-sequence $h_{\mathrm{bk}}(k)$ with the Kaiser-windowing-sequence with $\alpha = 9$

2.3.4 How big is the minimal number N_{V} of delay-elements necessary for the realisation of the causal digital FIR low-pass?