Problem 3.5

Given is the system function:

$$H_{\mathrm{L}}(p_1) = \frac{\omega_{\mathrm{T}}^2}{p_1^2 + \sqrt{2}\omega_{\mathrm{T}} p_1 + \omega_{\mathrm{T}}^2} \quad \text{with} \quad p_1 = \sigma_1 + \mathrm{j}\omega_1$$

of an analogue Butterworth-Low-Pass 2nd order with the 3dB-frequency ω_{T}. This low-pass shall be the reference system for an analogue high-pass, whos system function $H_{\mathrm{Lt}}(p)$ is related to the system function $H_{\mathrm{L}}(p_1)$ over the frequency transformation

$$p_1 = p_1(p) \quad \text{with} \quad p = \sigma + \mathrm{j}\omega$$

3.5.1 Give the function $p_1 = p_1(p)$ for the case that the wanted analogue high-pass has a 3dB frequency ω_{H}, which is k times ($k > 0$ and real) greater than ω_{T}.

3.5.2 Determine the corresponding system function $H_{\mathrm{Lt}}(p)$ of the analogue high-pass.

3.5.3 Determine the related function $\omega_1 = g_\omega(\omega)$, which can be derived from the function found in 3.5.1.

3.5.4 Determine the magnitude-characteristic $|H_{\mathrm{Ft}}(\omega)| = |H_{\mathrm{Lt}}(p = \mathrm{j}\omega)|$ of the wanted high-pass.

Problem 3.6

Given is an analogue Chebyshev low-pass second order.

3.6.1 Give the general equation for the approximating function $A(\omega)$ for the low-pass.

3.6.2 Develop the equation $A(\omega)$ of the Chebyshev low-pass in a polynomial.

3.6.3 Determine the poles of the function $A_{\mathrm{L}}(p)$.

3.6.4 Determine the poles of the system function $H_{\mathrm{L}}(p)$ of the low-pass and give $H_{\mathrm{L}}(p)$.

3.6.5 Using the impulse-invariance-method, the analogue low-pass shall be transformed in a digital low-pass with a sampling- and clock-frequency $\omega_{\mathrm{a}} = 2\pi/T_{\mathrm{a}}$. Determine the system function $H_{\mathrm{z}}(z)$ of the related digital low-pass.

3.6.6 Sketch a canonical structure of the digital low-pass and give the multiplying coefficients.

Problem 3.7

Given is the system function:

$$H_{\mathrm{L}}(p) = \frac{\omega_{\mathrm{T}}^2}{p^2 + \sqrt{2}\omega_{\mathrm{T}} p + \omega_{\mathrm{T}}^2}$$

of a stable analogue Butterworth-Low-Pass 2nd order with the 3dB-frequency ω_{T}. Using the impulse-invariance-method the analogue low-pass shall be transformed in a digital low-pass with a sampling- and clock-frequency $\omega_{\mathrm{a}} = 2\pi/T_{\mathrm{a}}$.

3.7.1 Determine the system function $H_{\mathrm{z}}(z)$ of the related digital low-pass.

3.7.2 Determine the filter-coefficients a_m and b_n of the related digital low-pass.

Problem 3.8

A given realisable analogue reference system with the system function $H_\mathrm{L}(p_\mathrm{k})$ can be transformed in a digital system by the transformation $H_\mathrm{z}(z) = H_\mathrm{L}(p_\mathrm{k}(z))$ with:

$$p_\mathrm{k} = \frac{2}{T_\mathrm{a}} \cdot \frac{z-1}{z+1} = p_\mathrm{k}(z)$$

with

$$p_\mathrm{k} = \sigma_\mathrm{k} + \mathrm{j}\omega_\mathrm{k} \quad \text{and} \quad z = \mathrm{e}^{pT_\mathrm{a}} = \mathrm{e}^{(\sigma+\mathrm{j}\omega)T_\mathrm{a}}$$

3.8.1 Show that the unit-circle $|z| = 1$ of the z-plane is mapped in the $\mathrm{j}\omega_\mathrm{k}$-axis of the p_k-plane.

3.8.2 Show that the $\mathrm{j}\omega$-axis of the p-plane is mapped onto the $\mathrm{j}\omega_\mathrm{k}$-axis of the p_k-plane.

3.8.3 Determine the frequency ω_k as function of ω and draw the function for $0 \leq \omega \leq \omega_\mathrm{a}$.

3.8.4 Draw the coordinate systems of the p-plane, the z-plane and the p_k-plane with the nomenclature of the corresponding abscissas and ordinates and denote the $\mathrm{j}\omega$-axis in the range $0 \leq \omega \leq \omega_\mathrm{a}$ of the p-plane. Input the points $p_n = \mathrm{j}n\omega_\mathrm{a}/4$ with $n = 0(1)4$ and the points $p_m = \mathrm{j}m\omega_\mathrm{a}/4$ with $m = 0(1)4$. Input the same points at the corresponding positions in the the z-plane and the p_k-plane.

Problem 3.9

Given is a Butterworth low-pass second order with system function:

$$H_\mathrm{L}(p) = \frac{\omega_\mathrm{g}^2}{p^2 + \sqrt{2}\omega_\mathrm{g} p + \omega_\mathrm{g}^2}$$

3.9.1 Give the system function $H_\mathrm{z}(z)$ of a digital filter, which has been derived from the given analogue reference system by the method of the bilinear z-transformation.

Problem 3.10

Given is the system function:

$$H_\mathrm{z}(z) = \frac{a_1 z}{b_2 z + b_0 z^2} \quad \text{with} \quad z = \mathrm{e}^{pT_\mathrm{a}}$$

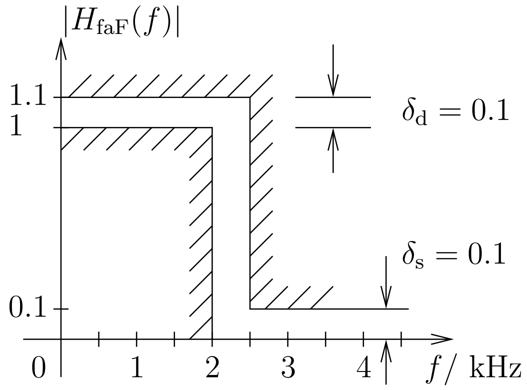
of a digital filter, which has been derived from the system function $H_\mathrm{L}(p_\mathrm{k} = \sigma_\mathrm{k} + \mathrm{j}\omega_\mathrm{k})$ of an analogue reference system.

3.10.1 Determine the system function $H_\mathrm{L}(p_\mathrm{k})$ of the analogue reference system.

3.10.2 Which kind of analogue filter is described by the reference system?

Problem 3.11

Given is the tolerance scheme of the magnitude characteristic $|H_\mathrm{faF}(f)| = |H_\mathrm{z}(z = \mathrm{e}^{\mathrm{j}2\pi f T_\mathrm{a}})|$ of a digital IIR low-pass beneath. This low-pass has been developed from an analogue reference system $H_\mathrm{L}(p_\mathrm{k})$ applying the method of the bilinear z-transformation. The sampling- and clock frequency of the digital system is equal to $f_\mathrm{a} = 8$ kHz.



3.11.1 Draw the corresponding tolerance scheme of the magnitude characteristic $|H_\mathrm{fF}(f_\mathrm{k})| = |H_\mathrm{L}(p_\mathrm{k} = \mathrm{j}2\pi f_\mathrm{k})|$ of the analogue reference system, inserting the corner-frequencies f_kd and f_ks.