Fachgebiet Nachrichtentechnische Systeme

Network Theory 3
Advanced Digital Filters

Chapter 6
Digital Signal Processors and its Applications

6.1 DSP Overview

DSP‘s typically exhibit a speed optimised design:

- Reduced set of instructions
- Several internal data busses working parallel
- No pipelines - one or more instructions are performed in one clock cycle
- Fixed and Floating Point DSP‘s are available
- ALU and MAC unit, Interfaces and DMA are optimised for real-time operation
- The same holds for serial data links, communication ports, cache

Example: Sharc DSP

- 40 bit IEEE floating point math or 80 bit integer
- 32 bit fixed-point multipliers, 64 bit product, 80 bit accumulation
- Circular buffer adressing for 32 buffers
- DMA allows zero-overhead background transfer at full clock rate
- Performance: 66 Mhz/198 MFLOPs to 400 Mhz/2400 MFLOPS
- Pricing: Down to 1$ per 300 MFLOPS

6.1 DSP Overview

<u>Other measures (for fixed processing rate)</u>

- Real-time operating systems
- No multitasking
- Special MAC instructions
- Fast loop processing
- Multiple instructions in a cycle like: 1 Multiplication + 1 addition + 1 substraction + 2 RAM fetches + Postincrements/decrements

<u>Application areas</u>

- Automotive (Radar,Ultrasound)
- Consumer/Entertainment (HD and SD audio and video)
- Fire and Security (Fire and intrusion detection by video cameras)
- Medicine (3D medical imaging)
- Security (Scanners)
- Mobile Radio/Communications (Processing in base stations, mobile phones)
- Ships (Sonar)
- Aircraft and satellites (Radar applications for remote sensing)

6.2 Audio Signal Processing Applications

Examples of analog audio signal processing

- Filters of high order and low phase distortions
- Noise reduction and dynamic compression
- Several effects like echo generation or echo suppression

- Multichannel audio (Surround encoding and decoding)

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Chapter 7
Digital Filters for Audio Applications

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7 Digital Audio Filters

Application areas

- Studio recording, mastering, re-mastering
- Live performances
- Sub-woofer equalisation

Filter types

- HP, LP shelving filters
- BP/BS filters
- Octave/Terz filters
- Filter banks
- Dynamic filters
- Feedback destroyers

7 Digital Audio Filters

Octave filters

- Show the same relative bandwidth
- In a filter bank center frequencies are on a logarithmic grid
- Setting all filters to the peak level gives allpass behaviour
- The center frequency of one filter is the double of the next filter

$$\omega_0 = \sqrt{2}\omega_- \quad \omega_+ = \sqrt{2}\omega_0 \text{ or } \omega_0 = \sqrt{\omega_- \omega_+}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{2\omega_- - \omega_-}{\sqrt{2}\omega_-} = 1/\sqrt{2}$$

Terz filters

- Show the same relative bandwidth (smaller than octave filters)
- Separation of an octave filter in 3 parts give a Terz filter
- The center frequency of one filter is the third root of 2 of the next filter

$$\omega_0 = \sqrt[6]{2}\omega_- \quad \omega_+ = \sqrt[6]{2}\omega_0 \text{ or } \omega_0 = \sqrt{\omega_- \omega_+}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{\sqrt[3]{2}\omega_- - \omega_-}{\sqrt[6]{2}\omega_-} = 0.2316$$

7 Digital Audio Filters - Design

- Recursive filters of second to fourth order are applied for speed reasons
- LP has therefore 2 up to 4 poles
- Typically a Butterworth filter type is applied (with poles on half a circle)
- For normalised system function of a recursive LP, HP, BP and BS filters of 2nd order hold:

$$H_{NLP}(P) = \frac{1}{1 + \frac{1}{Q}P + P^2} \quad \text{with } Q = 1/\sqrt{2} \text{ for Butterworth type}$$

$$H_{NHP}(P) = \frac{P^2}{1 + \frac{1}{Q}P + P^2}$$

$$H_{NBP}(P) = \frac{P/Q}{1 + P/Q + P^2} \quad \text{with } Q = \frac{\Delta\omega}{\omega_0}$$

$$H_{NBS}(P) = \frac{P^2 - 1}{1 + P/Q + P^2}$$

7 Digital Audio Filters - Design

LP shelving filter design (first order)

- Onset for amplifying low frequencies is to add a LP to an allpass
- This results in amplification of 1+ A_0 below a certain fixed cut-off frequency
- Amplification rises about 6dB/octave
- Onset for damping of low frequencies is to use inversed system function

$$H_{NLP+}(P) = 1 + \frac{A_0}{1+P} = \frac{1+A_0+P}{1+P} \text{ with cut-off frequency } \omega_g = p / P$$

$$H_{NLP-}(P) = \frac{1+P}{1+A_0+P}$$

LP Shelving filter design details

- The inversion ensures a stable cut-off frequency also for damping
- It exchanges pole and zero locations
- This is not given for negative A_0 value
- For Amplication of 12 dB/octave below cut-off a filter with 2 poles and 2 zeroes is needed
- For pole and zero of first order filter holds: $P_0 = -1 - A_0$ and $P_\infty =$ -1

7 Digital Audio Filters - Design

HP shelving filter design (first order)

- Onset for amplifying low frequencies is to add a HP to an allpass
- This results in amplification of 1+ A_0 above a certain fixed cut-off frequency
- Amplification rises about 6dB/octave
- Onset for damping of high frequencies is to use inversed system function

$$H_{NHP+}(P) = 1 + \frac{A_0 P}{1+P} = \frac{1+(A_0+1)P}{1+P} \text{ with cut-off frequency } \omega_g = p / P$$

$$H_{NHP-}(P) = \frac{1+P}{1+(A_0+1)P}$$

HP Shelving filter design details

- The inversion ensures a stable cut-off frequency also for damping and exchanges poles and zeros
- This is not given for negative A_0 value
- For Amplication of 12 dB/octave below cut-off a filter with 2 conjugated complex poles and 2 zeroes is needed
- For pole and zero of first order filter holds: $P_0 = -1/(1+A_0)$ and $P_\infty =$ -1

7 Digital Audio Filters - Design

BP filter design (first order)

- Onset for amplifying frequencies inside of a band is to add a BP to an allpass
- This results in an amplification in the center of the band
- The peak value of amplification is determined by 1 + A_0
- Onset for damping of pass-band frequencies is to use the inversed system function

$$H_{NBP+}(P) = 1 + \frac{A_0 P / Q}{1 + P/Q + P^2} = \frac{1 + (A_0 + 1) P / Q + P^2}{1 + P/Q + P^2}$$ with center frequency $\omega_0 = p / P$ and $\Delta\Omega = 1/Q = \Delta\omega / \omega_0$

$$H_{NBP-}(P) = \frac{1 + P/Q + P^2}{1 + (A_0 + 1) P / Q + P^2}$$

BP filter design details

- The inversion ensures a stable cut-off frequency also for damping and exchanges poles and zeros
- This is not given for negative A_0 value
- The 2 poles and 2 zeros are of the second order BP filter are located on the unit circle

7 Digital Audio Filters – The transform

- Now the bi-linear transform is used for moving the system function to the z-domain
- The impulse-invariant method cannot work due to low filter orders
- Therefore the transform to z-domain for a LP and a BP is achieved by:

$P = \frac{p}{\omega_g} = \cot(\pi \frac{\omega_g}{\omega_a}) \frac{z-1}{z+1}$ in combination with $H_{NLP+}(P) = \frac{A_0 + 1 + P}{1+P}$ leads to

$$H_{NLP+}(z) = \frac{1 + A_0 + \cot(\pi \frac{\omega_g}{\omega_a}) \frac{z-1}{z+1}}{1 + \cot(\pi \frac{\omega_g}{\omega_a}) \frac{z-1}{z+1}} = \frac{(z+1)(1+A_0) + \cot(\pi \frac{\omega_g}{\omega_a})(z-1)}{z + 1 + \cot(\pi \frac{\omega_g}{\omega_a})(z-1)}$$

$$= \frac{1 + A_0 - \cot(\pi \frac{\omega_g}{\omega_a}) + (1 + A_0 + \cot(\pi \frac{\omega_g}{\omega_a}))z}{1 - \cot(\pi \frac{\omega_g}{\omega_a}) + (1 + \cot(\pi \frac{\omega_g}{\omega_a}))z} = \frac{\frac{1 + A_0 - \cot(\pi \frac{\omega_g}{\omega_a})}{1 + \cot(\pi \frac{\omega_g}{\omega_a})} + \frac{1 + A_0 + \cot(\pi \frac{\omega_g}{\omega_a})}{1 + \cot(\pi \frac{\omega_g}{\omega_a})} z}{\frac{1 - \cot(\pi \frac{\omega_g}{\omega_a})}{1 + \cot(\pi \frac{\omega_g}{\omega_a})} + z}$$

$$= \frac{\frac{1 + A_0 + \cot(\pi \frac{\omega_g}{\omega_a})}{1 + \cot(\pi \frac{\omega_g}{\omega_a})} + \frac{1 + A_0 - \cot(\pi \frac{\omega_g}{\omega_a})}{1 + \cot(\pi \frac{\omega_g}{\omega_a})} z^{-1}}{1 + \frac{1 - \cot(\pi \frac{\omega_g}{\omega_a})}{1 + \cot(\pi \frac{\omega_g}{\omega_a})} z^{-1}}$$ to be compared with $H_Z(z) = \frac{a_0 + a_1 z^{-1}}{1 + b_1 z^{-1}}$

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$P = \frac{p}{\omega_0} = \cot(\pi \frac{\omega_0}{\omega_a}) \frac{z-1}{z+1}$ in combination with $H_{NBP+}(P) = \frac{1+(A_0+1)P/Q+P^2}{1+P/Q+P^2}$ leads with $A_0+1=B$ to:

$$H_{NBP+}(z) = \frac{1+\frac{B}{Q}\cot(\pi\frac{\omega_0}{\omega_a})\frac{z-1}{z+1}+(\cot(\pi\frac{\omega_0}{\omega_a})\frac{z-1}{z+1})^2}{1+\frac{1}{Q}\cot(\pi\frac{\omega_0}{\omega_a})\frac{z-1}{z+1}+(\cot(\pi\frac{\omega_0}{\omega_a})\frac{z-1}{z+1})^2} = \frac{(z+1)^2+\frac{B}{Q}\cot(\pi\frac{\omega_0}{\omega_a})(z^2-1)+\cot^2(\pi\frac{\omega_0}{\omega_a})(z-1)^2}{(z+1)^2+\frac{1}{Q}\cot(\pi\frac{\omega_0}{\omega_a})(z^2-1)+\cot^2(\pi\frac{\omega_0}{\omega_a})(z-1)^2}$$

$$= \frac{1-\frac{B}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a})+z(2-\cot^2(\pi\frac{\omega_0}{\omega_a}))+z^2(1+\frac{B}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a}))}{1-\frac{1}{Q}\cot(\pi\frac{\omega_g}{\omega_a})+\cot^2(\pi\frac{\omega_g}{\omega_a})+z(2-\cot^2(\pi\frac{\omega_g}{\omega_a}))+z^2(1+\frac{1}{Q}\cot(\pi\frac{\omega_g}{\omega_a})+\cot^2(\pi\frac{\omega_g}{\omega_a}))}$$

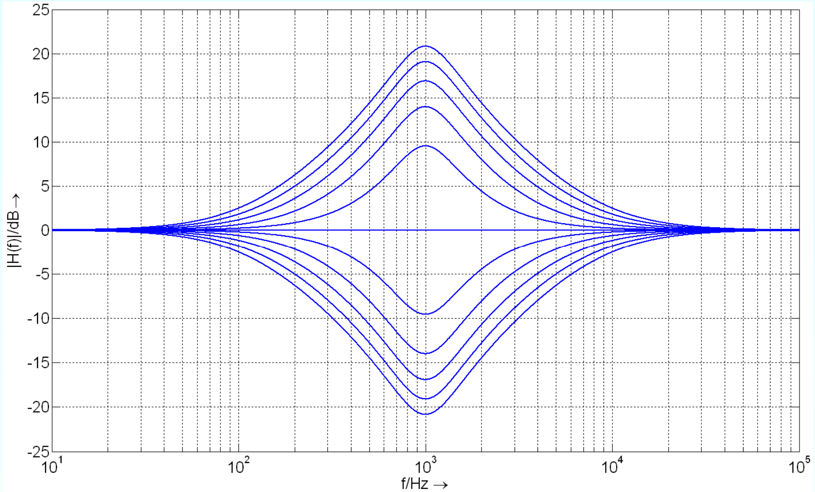
$$= \frac{\frac{1-\frac{B}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a})}{1+\frac{1}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a})}+\frac{(2-\cot^2(\pi\frac{\omega_0}{\omega_a}))}{1+\frac{1}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a})}z+\frac{(1+\frac{B}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a}))}{1+\frac{1}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a})}z^2}{\frac{1-\frac{1}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a})}{1+\frac{1}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a})}+\frac{2-\cot^2(\pi\frac{\omega_0}{\omega_a})}{1+\frac{1}{Q}\cot(\pi\frac{\omega_0}{\omega_a})+\cot^2(\pi\frac{\omega_0}{\omega_a})}z+z^2}$$

to be compared with $H_Z(z) = \frac{a_0+a_1z^{-1}+a_2z^{-2}}{1+b_1z^{-1}+b_2z^{-2}} = \frac{a_0z^2+a_1z+a_2}{z^2+b_1z+b_2}$

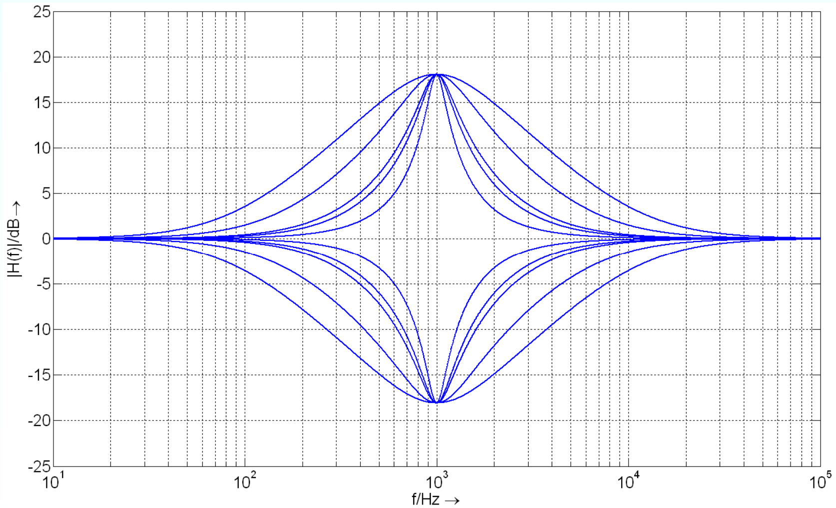
7 Digital Audio Filters

- As with LP‘s and HP‘s for attenuation inside of a certain frequency band poles and zeros are exchanged (i.e. numerator and denominator of the last fraction formula)
- For flexible/frequent changes of center frequency, amplification and bandwidth the filter coefficients have to be calculated each time according to previous formulas
- For easier calculation special parametric filter structures are used
- These use seperation of LP, HP, BP, BS filters into several allpasses

7 Digital Audio Filters



- Variation of amplification and attenuation



- Variation of Q

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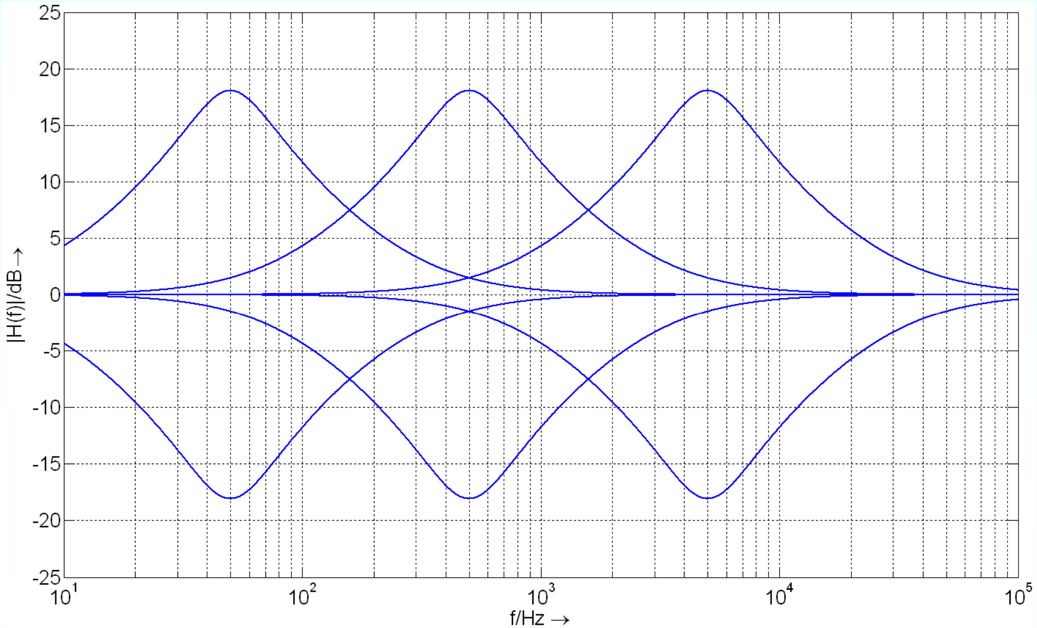

- Variation of center frequency

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7 Digital Audio Filters –FIR filter implementation

- Any FIR-filter difference equation can be implemented as a convolution of the input data stream with the finite discrete impulse response
- For high filter orders the DFT gives significant speed improvements
- Both the normal and the inverse DFT have to be applied
- DFT is often applied to complex data (in case of filtering 2 independant channels as in stereo application) using Re{..} and Im{..} operations
- For limited delay of the output (quick response) the input data stream has to be processed in partitions (of N values) according to:

$$s_n(k) = s(k) \cdot rect(\frac{k - nN - 0.5}{N}) \text{ with } s(k) = \sum_{n=0}^{\infty} s_n(k)$$

$$g(k) = s(k) * h(k) \text{ , } g(k) = \sum_{n=0}^{\infty} g_n(k) \text{ with } g_n(k) = s_n(k) * h(k)$$

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The following diagram shows the details of the method: