Problem 1.3 Given is the wanted transfer-function of an ideal digital high-pass:

$$H_{\mathrm{wF}}(\omega) = H_{\mathrm{bFHP}}(\omega) * \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_{\mathrm{a}})$$

For the baseband-function $H_{\mathrm{bFHP}}(\omega)$ the following holds:

$$H_{\mathrm{bFHP}}(\omega) = \mathrm{rect}\Big(\frac{\omega}{\omega_{\mathrm{a}}}\Big) - \mathrm{rect}\Big(\frac{\omega}{2\omega_{\mathrm{H}}}\Big)$$

with

$$0 < \omega_{\mathrm{H}} < \frac{\omega_a}{2}$$

A causal digital FIR high-pass shall be designed, whos magnitude characteristics $|H_{\mathrm{rF}}(\omega)|$ approximate the magnitude characteristics $|H_{\mathrm{wF}}(\omega)|$ of the given ideal digital high-pass.

1.3.1 Derive a general equation for the calculation of the values of the unweighted impulse sequence $h_{\mathrm{bk}}(k)$ of the causal digital FIR high-pass. Given are the high-pass frequency ω_{H}, the sampling- (and clock-) frequency ω_{a} and the number of filter coefficients $N = 2N_{\mathrm{f}} + 1$.

1.3.2 Determine the general equation for the calculation of the values of the weighted impulse sequence $h_{\mathrm{rk}}(k) = h_{\mathrm{wk}}(k) \cdot h_{\mathrm{bk}}(k)$ of the causal digital FIR high-pass, if the coefficients of the weighting function $h_{\mathrm{wk}}(k)$ are equal to a Blackman-sequence $h_{\mathrm{Bk}}(k)$.

1.3.3 Determine the values of the weighted impulse sequence $h_{\mathrm{rk}}(k)$ of the causal digital FIR high-pass and plot them over k. The following data for the realisation of the filter is given:

- Cut-off-frequency $f_{\mathrm{H}} = \omega_{\mathrm{H}}/2\pi = 20$ kHz
- Clock-frequency $f_{\mathrm{a}} = \omega_{\mathrm{a}}/2\pi = 100$ kHz
- Number of filter coefficients $N = 9$
- Weighting of the baseband-sequence $h_{\mathrm{bk}}(k)$ with the Blackman-windowing-sequence $h_{\mathrm{Bk}}(k)$

1.3.4 How big is the minimal number N_{V} of delay-elements necessary for the realisation of the causal digital FIR high-pass?

Problem 1.4

Given is a first order low-pass with system function:

$$H_{\mathrm{L}}(p) = \frac{\omega_{\mathrm{L}}}{p + \omega_{\mathrm{L}}}$$

1.4.1 Give the system function $H_{\mathrm{z}}(z)$ of a digital high-pass filter, which has been derived from the given analogue reference system by the method of the bilinear z-transformation.