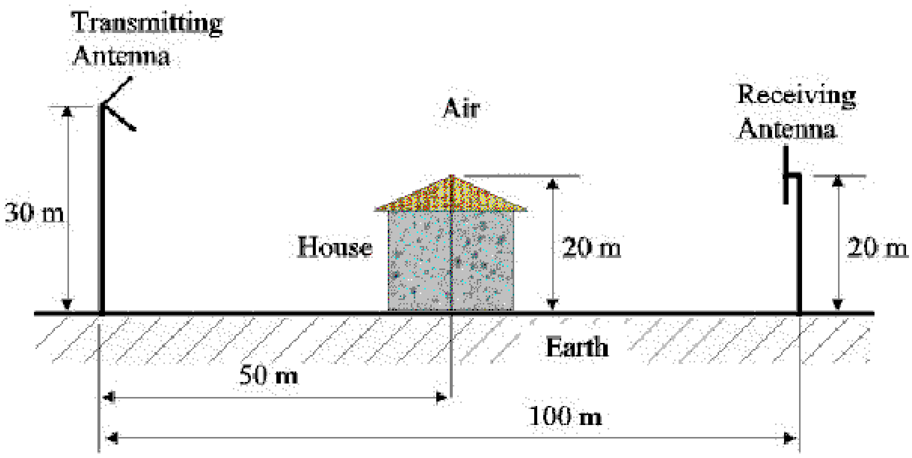
Wave Propagation In Far-Field Region

Problem 1

Prove that the power received by any type of antenna is always smaller than the power fed into the corresponding transmitting antenna irrespective of the gains of the two antennas.

Problem 2

The height above plane earth of a transmitting and receiving antenna are, respectively, 30 m and 20 m, as illustrated in the figure below. The ground separation of the two antennas is 100 m. Half way between the antennas is a house whose roof top is 20 m above ground. The transmitting antenna is assumed to be lossless and ideal. It radiates a harmonic electromagnetic wave of frequency $f = 3$ GHz towards the receiving antenna.



2.1 Is there an unobstructed line-of-sight path between the two antennas?

2.2 A round horn antenna with a gain of 30 dB relative to an isotropic radiator is used at the transmitting site. Find out whether the receiving antenna is in the far-field region of the transmitting antenna.

2.3 The transmitted signal power is now 1 mW, and the same antenna as the transmitting antenna is used at the receiver site. Determine the effective value of the electric field $E_{0,\text{eff}}$ and the received signal power P_R considering only the wave that reaches the receiving antenna by way of the direct line-of-sight path.

Solution of Problem 1

In far-field region we know that the available power at the receiving antenna, from the Friis formula, is

$$P_\mathrm{R} = \frac{P_\mathrm{T} G_\mathrm{T}}{4\pi d^2} \cdot A_\mathrm{R,eff} = \frac{P_\mathrm{T} G_\mathrm{T}}{4\pi d^2} \cdot \frac{\lambda^2 G_\mathrm{R}}{4\pi} = P_\mathrm{T} G_\mathrm{T} G_\mathrm{R} \left(\frac{\lambda}{4\pi d}\right)^2 , \tag{1}$$

with P_R, P_T, G_T, G_R, d and λ denoting received signal power, transmitted signal power, the gain of the transmitting antenna, the gain of the receiving antenna, the distance between the two antennas and the wavelength, respectively.

We assume that both antennas are identical, and therefore we have

$$G_\mathrm{T} = G_\mathrm{R} = G . \tag{2}$$

Theoretically, the gain of the transmitting and receiving antenna could be made arbitrarily large such that one could imagine that the received signal power becomes larger than the transmitted one which would be a "*perpetual motion machine*" producing more energy than was fed into. However, this is never the case.

Since the Friis formula is only valid for the far-field region, we must consider the condition

$$d \geq \frac{2D_0^2}{\lambda} , \tag{3}$$

where D_0 is the geometrical dimension of the antenna.

Figure 1: Examples of horn antennas.

In case of horn antennas, see Figure 1, which provide large gains, the geometrical area of the aperture of these antennas is

$$A_\mathrm{g} = \pi \frac{D_0^2}{4} . \tag{4}$$

Let us assume the most favorable situation of lossless and ideal antennas. Hence, the effective area A_eff is given by

$$A_\mathrm{eff} = A_\mathrm{g} = \pi \frac{D_0^2}{4} , \tag{5}$$

and with

$$A_{\text{eff}} = \frac{\lambda^2}{4\pi} G \quad \Rightarrow \quad G = A_{\text{eff}} \frac{4\pi}{\lambda^2} \tag{6}$$

the gain of both antennas is

$$G \overset{(6)}{=} A_{\text{eff}} \frac{4\pi}{\lambda^2} \overset{(5)}{=} \frac{\pi D_0^2}{4} \cdot \frac{4\pi}{\lambda^2} = \left(\frac{\pi D_0}{\lambda}\right)^2 . \tag{7}$$

Considering that D_0 must fulfill the inequality of the far-field condition (3), one can get

$$D_0^2 \le \frac{d\lambda}{2} , \tag{8}$$

with (7) yields

$$G \le \pi^2 \frac{d}{2\lambda} , \tag{9}$$

and finally

$$P_{\text{R}} \le P_{\text{T}} \left(\pi^2 \frac{d}{2\lambda}\right)^2 \left(\frac{\lambda}{4\pi d}\right)^2 = P_{\text{T}} \left(\frac{\pi}{8}\right)^2 \le P_{\text{T}} . \tag{10}$$

Solution of Problem 2

2.1 An unobstructed line-of-sight path exists if the first Fresnel ellipse (or zone) is free of any obstacles. This first Fresnel ellipse is defined as the location of all points that fulfill the condition

$$\Delta x = l_1 + l_2 - d = \frac{\lambda}{2} . \tag{11}$$

Therefore, if the sum of the distances l_{H1} and l_{H2} from the transmitting antenna to the roof top and from the roof top to the receiving antenna, respectively, are larger than $d + \frac{\lambda}{2}$, the first Fresnel ellipse is free of any obstacle and consequently, there is a true line-of-sight (LOS) path. The condition can be written as

$$l_{\text{H1}} + l_{\text{H2}} - d > \frac{\lambda}{2} , \tag{12}$$

where d is the distance of the direct path.

Using the given geometry, the distances d, l_{H1}, l_{H2} and the wavelength λ are, respectively,

$$d = \sqrt{(h_{\text{T}} - h_{\text{R}})^2 + s^2} \approx 100.5 \text{ m} , \tag{13}$$

$$l_{\text{H1}} = \sqrt{(h_{\text{T}} - h_{\text{H}})^2 + s_{\text{H}}^2} \approx 51 \text{ m} , \tag{14}$$

$$l_{\text{H2}} = \sqrt{(h_{\text{H}} - h_{\text{R}})^2 + (s - s_{\text{H}})^2} = 50 \text{ m} , \tag{15}$$

$$\lambda = \frac{c}{f} \approx 0.1 \text{ m} , \tag{16}$$

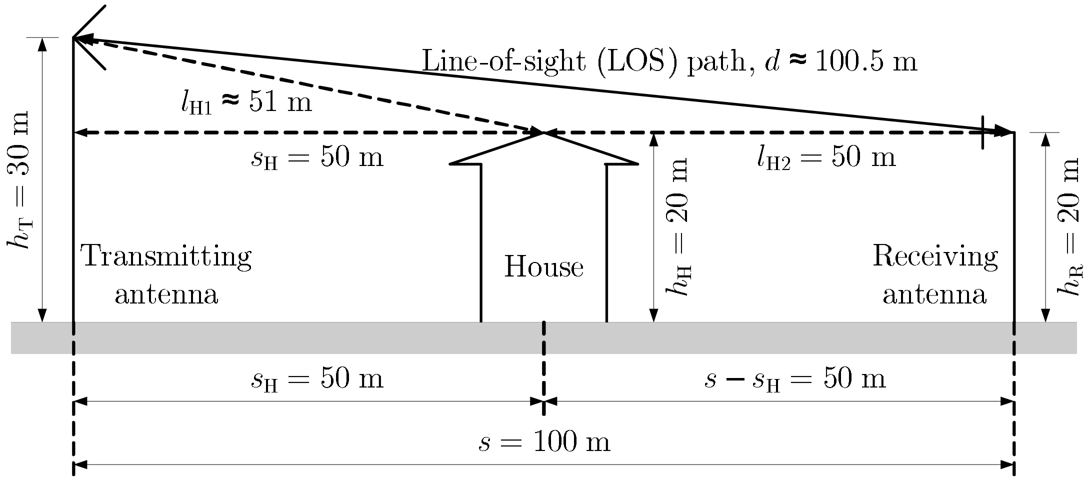


Figure 2: Geometry of Problem 2.

where $h_{\mathrm{T}} = 30$ m, $h_{\mathrm{R}} = 20$ m, $h_{\mathrm{H}} = 20$ m, $s_{\mathrm{H}} = 50$ m and $s = 100$ m are the height of the transmitting antenna, the height of the receiving antenna, the height of the roof top, the ground separation of the transmitting antenna to the roof top and the ground distance from the transmitting to the receiving antenna, respectively. See Figure 2. Hence, we obtain

$$l_{\mathrm{H1}} + l_{\mathrm{H2}} - d \;\approx\; 0.5 \text{ m} \;>\; \frac{\lambda}{2} \;=\; 0.05 \text{ m} . \qquad (17)$$

From that, we can conclude that there is indeed a line-of-sight path between the transmitting and receiving antenna.

2.2 The condition for the far-field region is

$$d \;\geq\; \frac{2D_0^2}{\lambda} , \qquad (18)$$

with D_0 denoting the largest dimension of the transmitting antenna. Since the antenna is lossless and ideal, we can assume that the effective area A_{eff} is identical with the geometrical area A_{g}

$$A_{\mathrm{T,eff}} \;=\; G_{\mathrm{T}}\frac{\lambda^2}{4\pi} \;\stackrel{!}{=}\; A_{\mathrm{g}} \;=\; \pi\frac{D_0^2}{4} . \qquad (19)$$

Rearranging this and considering $\lambda \approx 0.1$ m and $G_{\mathrm{T}} = 30$ dB $= 1000$ yields

$$D_0 \;=\; \sqrt{\frac{G_{\mathrm{T}}\lambda^2}{\pi^2}} \;\approx\; 1.01 \text{ m} . \qquad (20)$$

Checking the far-field condition (18), we obtain

$$d \;\approx\; 100.5 \text{ m} \;>\; \frac{2D_0^2}{\lambda} \;\approx\; 20.4 \text{ m} , \qquad (21)$$

which proves that the receiving antenna is in the far-field zone of the transmitting antenna.

2.3 The relation between power density (or effective value of the magnitude of the Poynting vector) P' and the effective electric field strength $E_{0,\text{eff}}$ is given by

$$P' = \frac{E_{0,\text{eff}}^2}{Z_0} , \tag{22}$$

where $Z_0 = 120\pi\ \Omega \approx 377\ \Omega$ is the characteristic impedance of free space. Since the power density is also

$$P' = \frac{P_\text{T} G_\text{T}}{4\pi d^2} , \tag{23}$$

we can calculate the effective value of the electric field at the transmitting antenna

$$E_{0,\text{eff}} = \sqrt{\frac{P_\text{T} G_\text{T} Z_0}{4\pi d^2}} = \frac{\sqrt{30 P_\text{T} G_\text{T}}}{d} \approx 54.5\ \text{mV/m} . \tag{24}$$

The received signal power with $G_\text{T} = G_\text{R} = 30\ \text{dB} = 1000$ and $A_{\text{T,eff}} = A_{\text{R,eff}} \stackrel{!}{=} A_\text{g}$ is then

$$P_\text{R} = \frac{E_{0,\text{eff}}^2}{Z_0} A_{\text{R,eff}} = \frac{E_{0,\text{eff}}^2}{Z_0} \cdot \frac{\lambda^2 G_\text{R}}{4\pi} = \left(\frac{E_{0,\text{eff}} \cdot \lambda}{2\pi}\right)^2 \frac{G_\text{R}}{120\Omega} \approx 6.27\ \mu\text{W} . \tag{25}$$