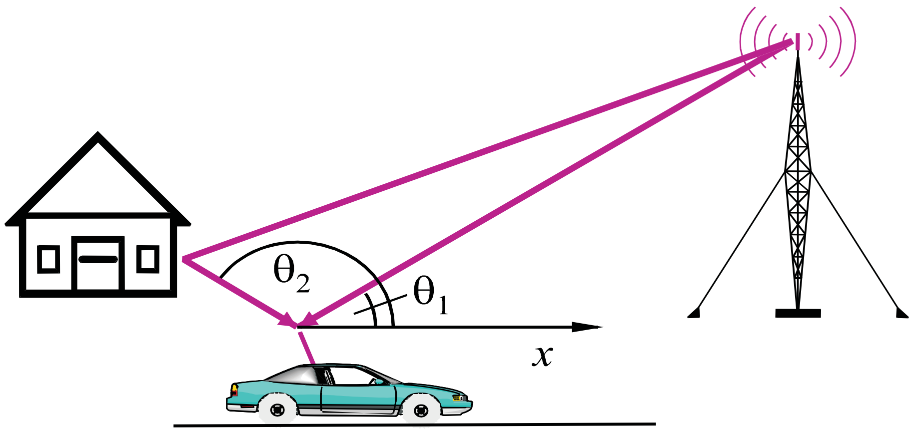
Linear Time-Variant Multipath Channel

Problem 1

A mobile station receives electromagnetic plane waves from a base station while it is moving with the constant velocity v as depicted in the figure below. It is assumed that the distance between mobile and base station is much larger than the location change of the mobile station caused by the movement.



The **ideal** linear time-invariant multipath channel can be expressed as

$$h(\tau) = \sum_{i=1}^{N} A_i \delta(\tau - \tau_i) \ .$$

1.1 Due to the movement of the mobile station, the time delays τ_i change and hence, the channel becomes time-variant. Calculate the linear time-variant impulse response $h(t, \tau)$ of the channel and sketch it for $N = 2$.

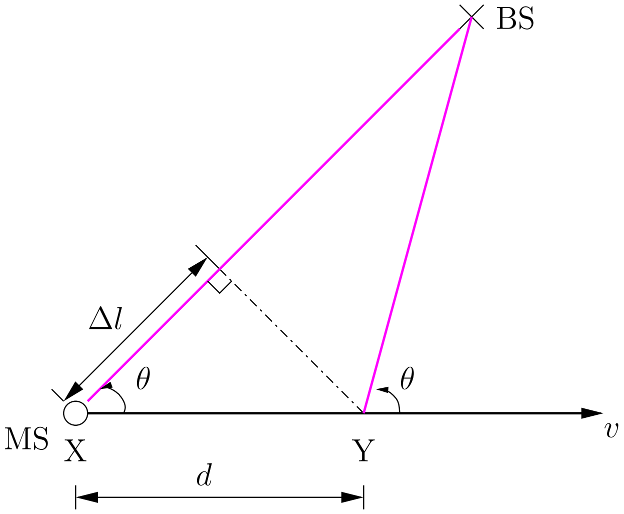
1.2 Now assume that the base station sends the signal $x(t) = \delta(t - t_0)$ to the mobile station. Calculate and sketch the received signal $y(t)$ at the mobile station.

1.3 Calculate the time-variant transfer function $T(t, \omega)$ with the relation $T(t, \omega) \overset{\mathcal{F}^{-1}}{\bullet\!\!-\!\!\circ} h(t, \tau)$.

1.4 Calculate the delay Doppler-spread function $S(\nu, \tau)$ and the output Doppler-spread function $H(\nu, \omega)$.

Remarks on Doppler Shift

Consider a mobile station (MS) moving at a constant velocity v, along a path segment having length d between points X and Y, while it receives signals from a remote base station (BS). See the following figure.



The difference in path lengths traveled by the wave from base station to the mobile station at points X and Y is $\Delta l = d\cos\theta = v\Delta t\cos\theta$, where Δt is the time required for the mobile station to travel from X to Y, and θ is assumed to be the same at points X and Y since the base station is assumed to be very far away. The phase change in the received signal due to the difference in path lengths Δl is therefore

$$\Delta\varphi = 2\pi\frac{\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta \tag{1}$$

and hence the apparent change in frequency, or *Doppler shift*, is given by f_d, where

$$f_d = \frac{1}{2\pi}\cdot\frac{\Delta\varphi}{\Delta t} = \frac{v}{\lambda}\cos\theta\ . \tag{2}$$

Equation (2) relates the Doppler shift to the mobile velocity and the spatial angle between the *direction of the motion of the mobile station* and the *direction of arrival of the wave.* It can be seen from Equation (2) that if the mobile is moving toward the direction of arrival of the wave, the Doppler shift is *positive* (i.e., *the apparent received frequency is increased*), and if the mobile is moving away from the direction of arrival of the wave, the Doppler shift is *negative* (i.e., *the apparent received frequency is decreased*). Therefore, multipath components from a signal that arrive from different directions contribute to *Doppler spreading* of the received signal, thus increasing the signal bandwidth.

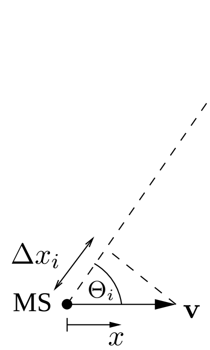
Solution of Problem 1

1.1 First, we consider only for the i-th path.

Within the time interval t, the mobile station comes closer to the base station by the distance

$$\Delta x_i(t) = vt \cdot \cos\Theta_i \ .$$

Hence, the plane wave of the i-th path from the base station arrives earlier at the mobile station by $\Delta\tau_i(t) = \Delta x_i(t)/c$.



Taking N paths into account and neglecting time dependence of N and A_i (actually the number of multipath components N and the amplitude of each path A_i are both time-variant) yields

$$\begin{aligned} h(t,\tau) &= \sum_{i=1}^{N} A_i \delta\big(\tau - \tau_i(t)\big) \\ &= \sum_{i=1}^{N} A_i \delta\Big(\tau - \big(\tau_i - \Delta\tau_i(t)\big)\Big) \\ &= \sum_{i=1}^{N} A_i \delta\left(\tau - \tau_i + \frac{v}{c}\cos\Theta_i \cdot t\right) \ . \end{aligned} \tag{1}$$

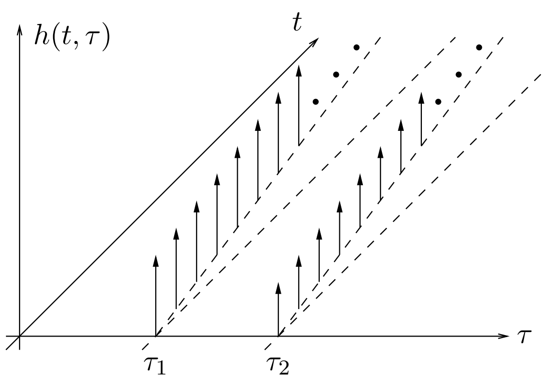


Figure 1: Example of channel impulse response $h(t,\tau)$ for $N = 2$ and real positive A_i .

1.2 During observation (t-axis) the channel (τ-axis) changes – the delay τ_i decreases with increasing t since the mobile station comes closer to the base station. We know from the lecture

$$\begin{aligned} y(t) &= h(t,\tau) * x(t) = h(t,\tau) * \delta(t-t_0) \\ &= \int_{-\infty}^{\infty} h(t,\tau)\delta(t-t_0-\tau)\mathrm{d}\tau = h(t,t-t_0) \\ &= \sum_{i=1}^{N} A_i \delta\left(t - t_0 - \tau_i + \frac{v}{c}\cos\Theta_i \cdot t\right) \ . \end{aligned} \quad (2)$$

Figure 2 illustrates the received signal $y(t)$ for $N=2$, where $t_1 = t_0 + \tau_1 - \frac{v}{c}\cos\Theta_1 \cdot t$ and $t_2 = t_0 + \tau_2 - \frac{v}{c}\cos\Theta_2 \cdot t$, respectively.

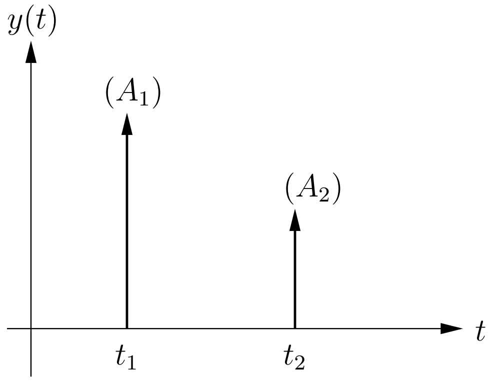


Figure 2: Example of received signal $y(t)$ for $N=2$.

1.3 With the time-variant transfer function

$$T(t,\omega) \quad \overset{\mathcal{F}^{-1}}{\bullet\!\!-\!\!\circ} \quad h(t,\tau) = \sum_{i=1}^{N} A_i \delta\left(\tau - \tau_i + \frac{v}{c}\cos\Theta_i \cdot t\right)$$

and the properties of FOURIER-transform $\delta(\tau) \overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} 1$ and $x(\tau-\tau_0) \overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} X(\omega)\mathrm{e}^{-\mathrm{j}\omega\tau_0}$, we have

$$T(t,\omega) = \sum_{i=1}^{N} A_i \mathrm{e}^{-\mathrm{j}\omega\tau_i} \mathrm{e}^{\mathrm{j}\omega\frac{v}{c}\cos\Theta_i \cdot t} \ . \quad (3)$$

1.4 With the delay Doppler-spread function

$$S(\nu,\tau) \quad \overset{\mathcal{F}^{-1}}{\bullet\!\!-\!\!\circ} \quad h(t,\tau) = \sum_{i=1}^{N} A_i \delta\left(\tau - \tau_i + \frac{v}{c}\cos\Theta_i \cdot t\right)$$

and the relation $\delta(at) = \frac{1}{|a|}\delta(t)$, the properties of FOURIER-transform $\delta(t) \overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} 1$ and $x(t-t_0) \overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} X(\omega)\mathrm{e}^{-\mathrm{j}\omega t_0}$, we have

$$S(\nu,\tau) = \sum_{i=1}^{N} A_i \left| \frac{c}{v\cos\Theta_i} \right| \mathrm{e}^{\mathrm{j}\nu \frac{c}{v\cos\Theta_i}(\tau-\tau_i)} \ . \tag{4}$$

Similarly, with

$$H(\nu,\omega) \quad \stackrel{\mathcal{F}^{-1}}{\bullet\!\!-\!\!\circ} \quad T(t,\omega) = \sum_{i=1}^{N} A_i \mathrm{e}^{-\mathrm{j}\omega\tau_i} \mathrm{e}^{\mathrm{j}\omega\frac{v}{c}\cos\Theta_i \cdot t}$$

and the properties of Fourier-transform $1 \stackrel{\mathcal{F}}{\circ\!\!-\!\!\bullet} 2\pi\delta(\nu)$ and $x(t)\mathrm{e}^{\mathrm{j}\nu_0 t} \stackrel{\mathcal{F}}{\circ\!\!-\!\!\bullet} X(\nu-\nu_0)$, we have

$$H(\nu,\omega) = \sum_{i=1}^{N} A_i \mathrm{e}^{-\mathrm{j}\omega\tau_i} 2\pi\delta\left(\nu - \omega\frac{v}{c}\cos\Theta_i\right) \ . \tag{5}$$