Multipath Propagation

Problem 1

The signal

$$x(t) = \sum_{n=-\infty}^{\infty} a_n s_0(t - nT_\mathrm{s}) \ , \quad s_0(t) = \Lambda\left(\frac{2t}{T_\mathrm{s}} - 1\right)$$

with T_s denoting the symbol period and

n	-2	-1	0	1	2	3	4	5
a_n	0	0	1	0	1	0	0	0

is to be transmitted via the propagation channel described by its *deterministic* impulse response

$$h(t) = \delta(t) - \frac{1}{2}\delta(t - 2T_\mathrm{s}) \ .$$

1.1 Sketch $s_0(t)$ and $h(t)$.

1.2 Sketch the transmitted signal $x(t)$ in the interval $t \in [0, 4T_\mathrm{s}]$.

1.3 Sketch the received signal $y(t) = x(t) * h(t)$ in the interval $t \in [0, 5T_\mathrm{s}]$. What kind of problem arises to detect the transmitted symbols?

Problem 2

Now, the radio channel is considered to be *random*. Assume that the channel impulse response $h(\tau)|_{\tau=\tau_0}$ at the particular delay $\tau = \tau_0$ can be approximately modeled by a real Gaussian random variable with mean $\overline{h}(\tau_0)$ and variance $\sigma^2_{h(\tau_0)}$.

Please note that this is just a simplified model and does not reflect reality.

2.1 Determine and sketch the probability density function $f_h(h, \tau_0)$ of the channel impulse response at $\tau = \tau_0$.

2.2 Explain the meaning of $\overline{h}(\tau_0)$ and $\sigma^2_{h(\tau_0)}$.

2.3 Determine the first moment (expected value) $\mathrm{E}\{h(\tau_0)\}$ and the second moment $\mathrm{E}\{h^2(\tau_0)\}$.

2.4 Determine the *power delay profile* $P(\tau) = \mathrm{E}\{h^2(\tau)\}$ as a function of $\overline{h}(\tau)$ and $\sigma^2_{h(\tau)}$.

Solution of Problem 1

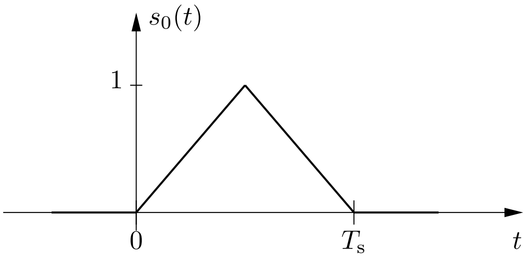
1.1 The signal

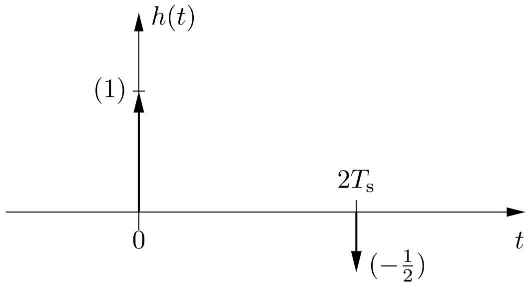
$$s_0(t) = \Lambda\left(\frac{2t}{T_\mathrm{s}} - 1\right) = \Lambda\left(\frac{2t - T_\mathrm{s}}{T_\mathrm{s}}\right) = \Lambda\left(\frac{t - T_\mathrm{s}/2}{T_\mathrm{s}/2}\right)$$

and the deterministic impulse response

$$h(t) = \delta(t) - \frac{1}{2}\delta(t - 2T_\mathrm{s})$$

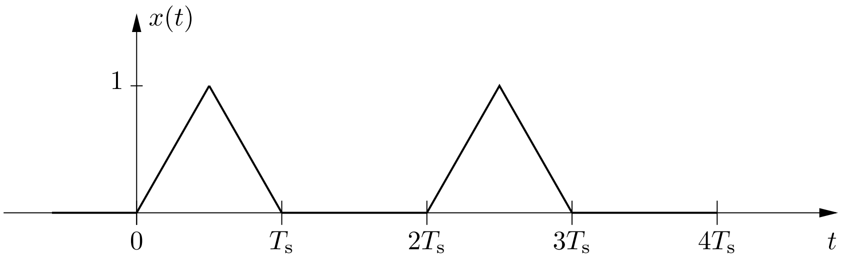
are sketched as follows.





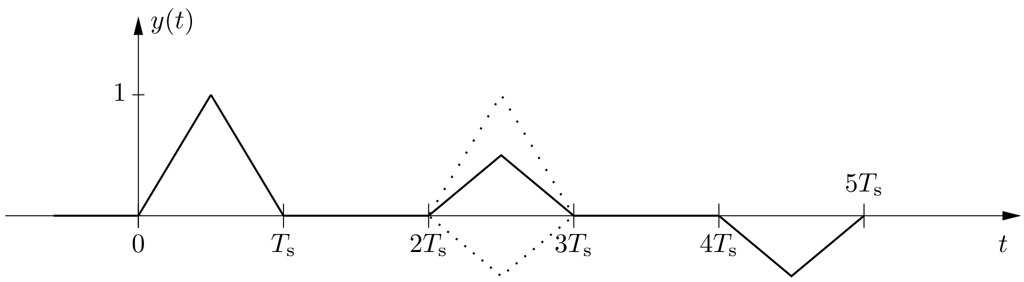
1.2 Transmitted signal:

$$x(t) = \sum_{n=-\infty}^{\infty} a_n s_0(t - nT_\mathrm{s}) = s_0(t) + s_0\left(t - 2T_\mathrm{s}\right) \ .$$



1.3 Received signal:

$$y(t) = x(t) * h(t) = x(t) - \frac{1}{2}x(t - 2T_\mathrm{s}) \ .$$

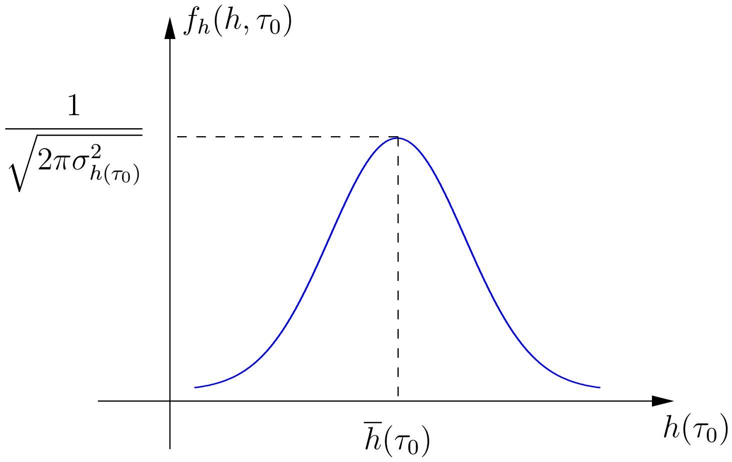


Since *intersymbol interference* (ISI) occurs, the received signal must be equalized to detect the transmitted symbols.

Solution of Problem 2

2.1 The probability density function $f_h(h,\tau_0)$ of the channel impulse response at $\tau = \tau_0$ is

$$f_h(h,\tau_0) = \frac{1}{\sqrt{2\pi\sigma^2_{h(\tau_0)}}}\, \mathrm{e}^{-\frac{1}{2}\left(\frac{h(\tau_0)-\overline{h}(\tau_0)}{\sigma_{h(\tau_0)}}\right)^2} .$$



2.2

$\overline{h}(\tau_0)$ $\rightarrow$ 'average' value of the channel impulse response at delay τ_0 ,

$\sigma^2_{h(\tau_0)}$ $\rightarrow$ 'uncertainty'/'fluctuation' of channel impulse response value at delay τ_0 .

2.3 First moment:

$$\mathrm{E}\left\{h(\tau_0)\right\} = \overline{h}(\tau_0) .$$

Second moment:

$$\mathrm{E}\left\{h^2(\tau_0)\right\} = \sigma^2_{h(\tau_0)} + \mathrm{E}^2\left\{h(\tau_0)\right\} = \sigma^2_{h(\tau_0)} + \overline{h}^2(\tau_0) .$$

2.4 The power delay profile of the random radio channel is

$$P(\tau) = \mathrm{E}\left\{h^2(\tau)\right\} = \sigma^2_{h(\tau)} + \mathrm{E}^2\left\{h(\tau)\right\} = \sigma^2_{h(\tau)} + \overline{h}^2(\tau) .$$