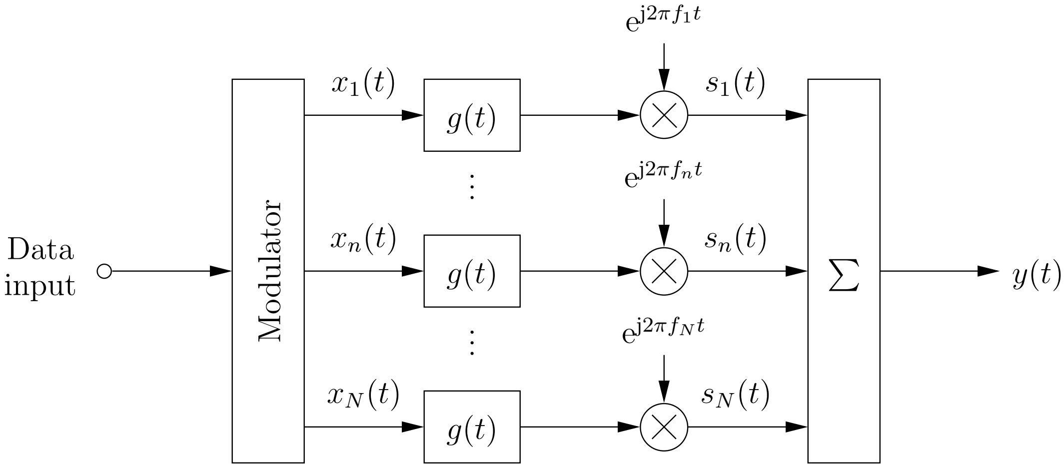
OFDM Modulation Technique

Problem 1

Orthogonal frequency division multiplexing (OFDM) is a modulation technique that has been suggested for various radio communication systems. Given is the following block diagram of an OFDM transmitter with N subcarriers.



For the sake of simplicity, it is assumed that on the n-th subcarrier ($n = 1, 2, \cdots, N$) only **one** symbol x_n is transmitted. Hence, it holds

$$x_n(t) = x_n \cdot \delta(t), \quad n = 1, 2, \cdots, N .$$

The impulse response $g(t)$ of the pulse shaping filter at the transmitter is given by

$$g(t) = \mathrm{rect}\left(\frac{t - NT/2}{NT}\right) = \begin{cases} 1 & , \quad 0 \le t \le NT \\ 0 & , \quad \text{else} \end{cases} .$$

The subcarrier frequencies f_n ($n = 1, 2, \cdots, N$) are

$$f_n = \frac{n}{NT} .$$

1.1 Determine the output signal $s_n(t)$ of the n-th subcarrier as a function of x_n.

1.2 Determine the FOURIER-transform $S_n(\omega) \overset{\mathcal{F}}{\bullet\!\!-\!\!\circ} s_n(t)$.

1.3 Sketch $|S_1(\omega)|$ and $|S_2(\omega)|$ - qualitatively - in the range $-\Delta\omega \le \omega \le 4\Delta\omega$ with all important points on abscissa and ordinate **in one diagram**.

Use the abbreviation $\Delta\omega = \frac{2\pi}{NT}$ and assume for this subproblem $x_1 = x_2 = 1$.

1.4 Determine the sampled signal $y_i = y(iT)$ for $1 \le i \le N$ at the output of the transmitter, and show that y_i ($i = 1, 2, \cdots, N$) is - apart from a constant scaling factor - identical to the result of the inverse discrete FOURIER-transform (IDFT) of x_n ($n = 1, 2, \cdots, N$).

Solution of Problem 1

1.1 $s_n(t)$ is the output signal of the pulse shaping filter on the n-th subcarrier which can be calculated as

$$\begin{aligned} s_n(t) &= [x_n(t) * g(t)] \cdot \mathrm{e}^{\mathrm{j}2\pi f_n t} = [(x_n \cdot \delta(t)) * g(t)] \cdot \mathrm{e}^{\mathrm{j}2\pi f_n t} \\ &= x_n \cdot [\delta(t) * g(t)] \cdot \mathrm{e}^{\mathrm{j}2\pi f_n t} \\ &= x_n \cdot g(t) \cdot \mathrm{e}^{\mathrm{j}2\pi f_n t} \\ &= x_n \cdot \mathrm{rect}\left(\frac{t - NT/2}{NT}\right) \cdot \exp\left(\mathrm{j}\,\frac{2\pi n}{NT}\,t\right) . \end{aligned}$$

1.2 With the following FOURIER-transform pairs

$$\begin{aligned} \mathrm{rect}\left(\frac{t}{NT}\right) &\overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} NT \cdot \mathrm{si}\left(\omega\frac{NT}{2}\right) \\ \mathrm{rect}\left(\frac{t - NT/2}{NT}\right) &\overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} NT \cdot \mathrm{si}\left(\omega\frac{NT}{2}\right) \cdot \exp\left(-\mathrm{j}\omega\frac{NT}{2}\right) \\ \mathrm{rect}\left(\frac{t - NT/2}{NT}\right) \cdot \exp\left(\mathrm{j}\,\frac{2\pi n}{NT}\,t\right) &\overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} NT \cdot \mathrm{si}\left(\left(\omega - \frac{2\pi n}{NT}\right)\frac{NT}{2}\right) \\ &\qquad \cdot \exp\left(-\mathrm{j}\left(\omega - \frac{2\pi n}{NT}\right)\frac{NT}{2}\right) , \end{aligned}$$

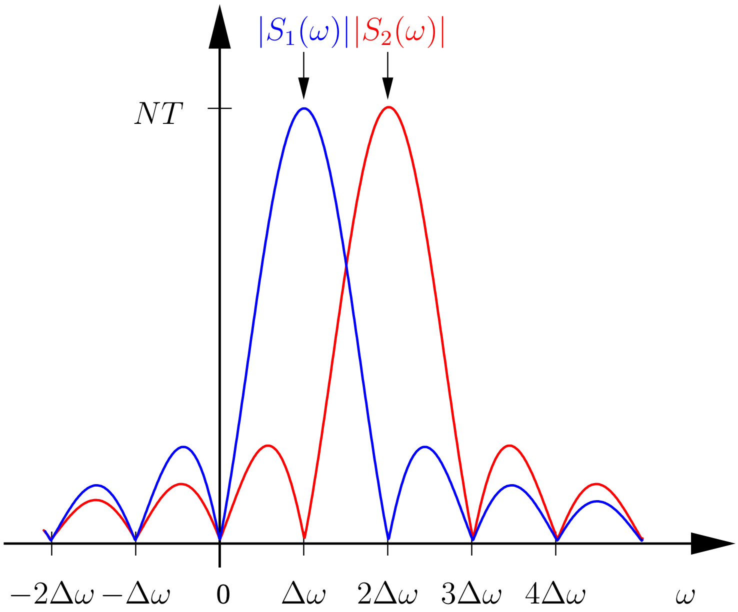
we can get

$$\begin{aligned} s_n(t) &= x_n \cdot \mathrm{rect}\left(\frac{t - NT/2}{NT}\right) \cdot \exp\left(\mathrm{j}\,\frac{2\pi n}{NT}\,t\right) \\ &\mathcal{F}\ \updownarrow \\ S_n(\omega) &= x_n \cdot NT \cdot \mathrm{si}\left(\left(\omega - n \cdot \frac{2\pi}{NT}\right)\frac{NT}{2}\right) \cdot \exp\left(-\mathrm{j}\left(\omega - n \cdot \frac{2\pi}{NT}\right)\frac{NT}{2}\right) . \end{aligned}$$

1.3 For the absolute value of $S_n(\omega)$ we have

$$\begin{aligned} |S_n(\omega)| &= |x_n| \cdot NT \cdot \left|\mathrm{si}\left(\left(\omega - n \cdot \frac{2\pi}{NT}\right)\frac{NT}{2}\right)\right| \\ &= |x_n| \cdot NT \cdot \left|\mathrm{si}\left(\pi\,\frac{\omega - n \cdot \Delta\omega}{\Delta\omega}\right)\right| \\ &= NT \cdot \left|\mathrm{si}\left(\frac{\pi}{\Delta\omega}\,(\omega - n\Delta\omega)\right)\right| . \end{aligned}$$

Hence, we can sketch $|S_1(\omega)|$ and $|S_2(\omega)|$ in the range $-\Delta\omega \leq \omega \leq 4\Delta\omega$ as follows.



1.4 The output signal $y(t)$ of the OFDM transmitter can be represented as

$$\begin{aligned} y(t) &= \sum_{n=1}^{N} s_n(t) \\ &= \sum_{n=1}^{N} x_n \cdot \mathrm{rect}\left(\frac{t - NT/2}{NT}\right) \cdot \exp\left(\mathrm{j}\,\frac{2\pi n}{NT} t\right) \ . \end{aligned}$$

Inserting $t = iT$ man can get

$$\begin{aligned} y_i &= y(iT) \\ &= \sum_{n=1}^{N} x_n \cdot \mathrm{rect}\left(\frac{iT - NT/2}{NT}\right) \cdot \exp\left(\mathrm{j}\,\frac{2\pi n}{NT} iT\right) \\ &= \sum_{n=1}^{N} x_n \cdot 1 \cdot \exp\left(\mathrm{j}\,\frac{2\pi n}{NT} iT\right) \\ &= \sum_{n=1}^{N} x_n \cdot \exp\left(\mathrm{j}\,\frac{2\pi}{N} ni\right) \ . \end{aligned}$$

The last equation tells us that y_i $(i = 1, 2, \cdots, N)$ is just the result of the inverse discrete FOURIER-transform (IDFT) of x_n $(n = 1, 2, \cdots, N)$.