Exercise 1:

1. Sketch the following functions

(a)

$$s(t) = A \cdot \mathrm{rect}\left(\frac{t-t_0}{T_0}\right)$$

(b)

$$\varepsilon(t-T_0)\cdot\sin(\omega_0 t+\varphi_0) \qquad \varphi_0 = \frac{2\pi}{3} \quad T_0 = \frac{2\pi}{\omega_0}$$

(c)

$$\Lambda\left(\frac{t}{T}-2\right)+\Lambda\left(\frac{t}{T}+2\right)$$

(d)

$$\mathrm{rect}\left(\frac{t}{T}+\frac{1}{2}\right)\cdot \mathrm{r}(-\frac{t}{T})$$

(e)

$$\mathrm{rect}\left(\frac{t}{2T_0}\right)\cdot\sin(\omega_0 t)$$

(f)

$$\Lambda\left(\frac{t}{T}\right)\cdot\Lambda\left(-\frac{t}{T}\right)$$

(g)

$$\varepsilon\left(-t+\frac{T}{2}\right)\cdot\mathrm{rect}\left(\frac{t}{2T}\right)$$

(h)

$$\Lambda\left(\frac{2t}{T}\right)+\mathrm{rect}\left(\frac{t}{2T}\right)$$

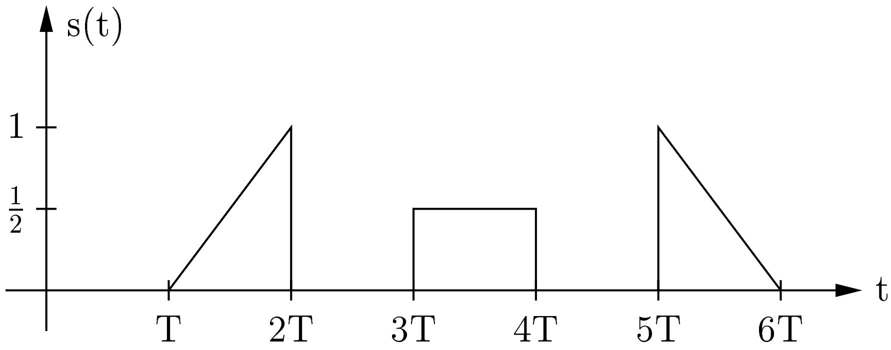
(i)

$$\sum_{n=-2}^{+2}\delta(t-nT)\cdot\Lambda\left(\frac{t}{2T}\right)$$

(j)

$$\delta(2t+T)-2\delta(-3t-2T); Note: \delta(at)=\frac{1}{|a|}\delta(t)$$

2. Determine the formula describing the following signal



Exercise 2:

A signal $s(t) = \sum_i A \cdot \Lambda\left(\frac{t - iT_0}{\frac{T_0}{2}}\right)$ is given with $A = 1V$ and $T_0 = 1ms$.

(a) Sketch the function $s(t)$
(b) Determine the Fourier coefficients a_n and b_n of the Fourier series of $s(t)$ for all $|n| < 6$

Exercise 3:

The signal $s_1(t)$ is given (T=1ms).

$$s_1(t) \quad = \quad 4 + 2 \cdot \sin\left(\frac{2\pi t}{T}\right) + 3 \cdot \sin\left(\frac{8\pi t}{T}\right) + 4 \cdot \cos\left(\frac{8\pi t}{T}\right)$$

(a) Write $s_1(t)$ in all other forms (trigonometric, polar, exponential resp.)
(b) Which harmonics are present ?
(c) Sketch the two sided magnitude-/phase spectrum (in the form of plots over f = 1/T
(d) Determine the RMS value

Exercise 4:

Given is a periodic voltage signal with the period T_0.

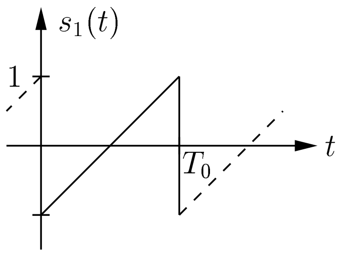
$$\underline{c}_n = \begin{cases} 1V \cdot e^{-j\frac{\pi}{4}} & \text{for} \quad n = -1 \\ 1V \cdot e^{+j\frac{\pi}{4}} & \text{for} \quad n = +1 \\ 0 & \text{else} \end{cases}$$

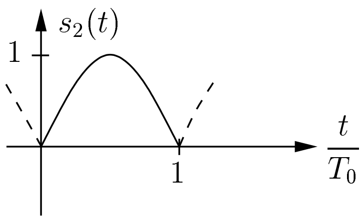
(a) Determine the Fourier coefficients a_n and b_n

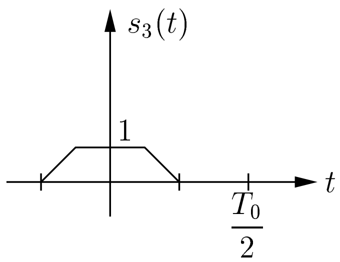
(b)Specify the expression for $s(t)$ including its dimension in trigonometric and polar form
(c) Specify the distortion factor

Exercise 5:

Determine the Fourier coefficients c_n, a_n and b_n for the following signals







Note: $s_3(t)$ is an even signal of the width $\frac{T_0}{2}$!

Exercise 6:

An even signal $s(t) = \sum_i 1V \cdot \Lambda\left(\frac{t - i \cdot T_0}{\frac{T_0}{2}}\right)$ with $T_0 = 1ms$ is given

(a) Determine the Fourier coefficients a_n and b_n

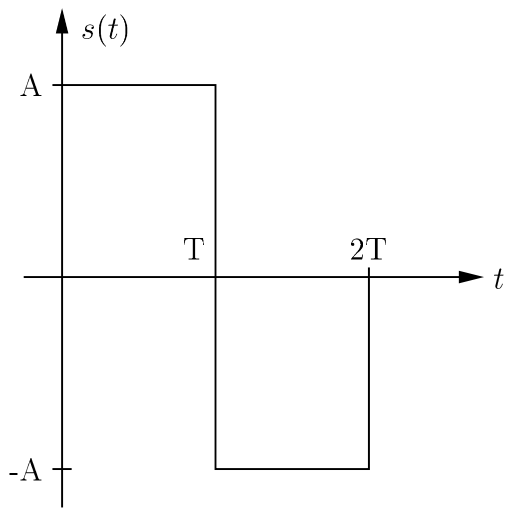
(b) Write down the Fourier transform $S_F(\omega)$ of a single pulse $\Lambda\left(\frac{t}{\frac{T_0}{2}}\right)$ and sketch this function

(c) Show, that for f=1kHz and 3kHz the relation $c_n = \frac{S_F(2\pi f)}{T_0}$ applies

Exercise 7:

Determine the Fourier transform $S_F(\omega)$ of the following signals

(a)



(b)

$$s(t) = \sin(\omega_0 t) + \cos(\omega_0 t)$$

Exercise 8:

Sketch and find $S_F(\omega)$ for:

$$s_1(t) = \exp\left(-2\frac{|t|}{T}\right)$$
$$s_2(t) = \text{rect}\left(\frac{t}{T} - \frac{1}{2}\right) - \Lambda\left(\frac{t - \frac{T}{2}}{\frac{T}{2}}\right)$$

Exercise 9:

Given is a discrete-time system with $g(k) = s(k-1) + \frac{1}{2} \cdot g(k-1)$

(a) Determine $g(k)$ for the case of a stimulation of the system with a unit step function
(b) Determine $H_z\left(e^{j\omega T_a}\right)$
(c) Write down $g(k)$ for a stimulation of the system with the unit impulse

Exercise 10:
Transform the signal $S_z(z)$ into the discrete-time signal $s(k)$

$$S_z(z) = 3 \cdot z^{-1} + 5 \cdot z^{-3} + 2 \cdot z^{-4}$$

Exercise 11:
Transform the signal $S_z(z)$ into the discrete-time signal $s(k)$

$$S_z(z) = \frac{1}{\left(z - \frac{1}{4}\right) \cdot \left(z - \frac{1}{2}\right)}$$

Exercise 12:

A signal $f(t)$ is given by the gaussian signal $f(t) = \exp\left\{-\left(\frac{t}{T}\right)^2\right\}$.

$f(t)$ is modulated by means of a cosine with a carrier frequency f_0 so that the signal $s(t) = f(t) \cdot \cos\left(2\pi \frac{t}{T_0}\right)$ is produced with $T >> T_0$.

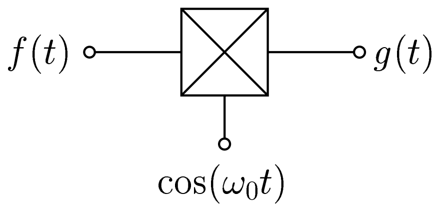
(a) Determine approximately the analytic spectrum $F^0(\omega)$
(b) Specify approximately the equivalent low pass spectrum $S_T(\omega)$

Exercise 13:
Given is a signal $s(t) = \cos(\omega_0 t) + 2 \cdot \sin(2 \cdot \omega_0 t) + 3 \cdot \cos\left(3 \cdot \omega_0 t + \frac{\pi}{2}\right)$

(a) Specify the Hilbert transform $\hat{s}(t)$
(b) Determine the equivalent low pass signal $f_T(t)$ for $f(t) = \cos(\omega_0 t) + 2 \cdot \sin(2 \cdot \omega_0 t)$ and write down the inphase component $u(t)$ und the quadrature component $v(t)$ for ω_0 as carrier frequency.

Exercise 14:
Deleted!

Exercise 15:
Given is the following modulator, which modulates the real valued signal $f(t)$



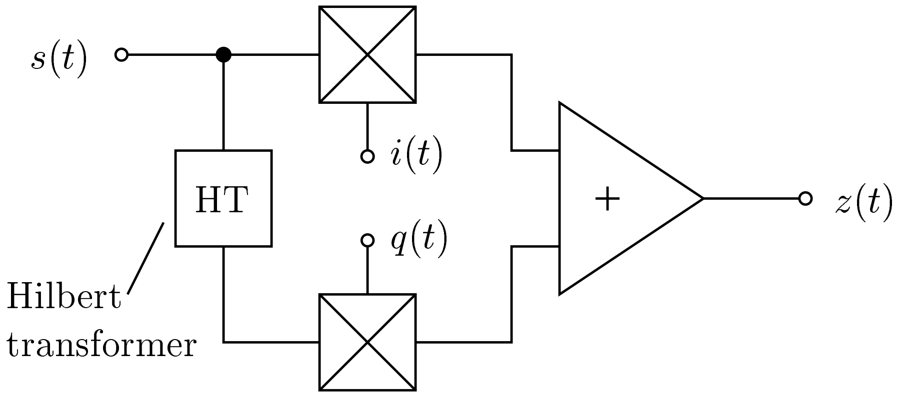
(a) Write down $F(\omega)$ as a function of $F^0(\omega)$
(b) Write down $F(\omega)$ as a function of $F_T(\omega)$
(c) Determine $G_T(\omega)$
(d) Sketch the functions $f(t)$, $g(t)$ and $g_T(t)$ for

$$f(t) = \Lambda\left(\frac{t}{T}\right) \quad \text{with} \quad \omega_0 = 5 \cdot \frac{2\pi}{T}$$

Exercise 16:
Given is a real signal $s(t)$. Prove, that

(a) the real part $R(\omega)$ of $S(\omega)$ is even and that the imaginary part $X(\omega)$ is uneven
(b) the magnitude of $S(\omega)$ is even and that the phase of $S(\omega)$ is uneven

Exercise 17:
Given is the following schematic with $s(t) = \cos(\omega_0 t + \varphi_0)$



(a) Determine the output signal $z(t)$, the inphase component $u(t)$ and the quadrature component $v(t)$ as a function of $i(t)$ and $q(t)$
(b) Determine $z(t)$ for $i(t) = A \cdot \cos(\omega_x t)$ and $q(t) = A \cdot \sin(\omega_x t)$
(c) Explain a possible application of this system!
(d) Determine $z_T(t)$ and $z^0(t)$ for case (b)
(e) Determine $z(t)$ and $z_T(t)$ for

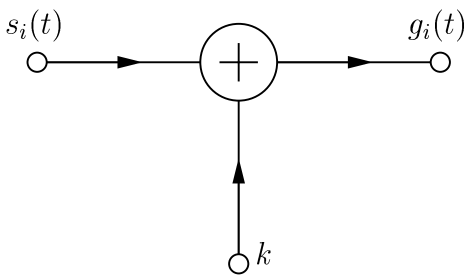
$$
\begin{aligned}
i(t) &= \sqrt{2} \cdot \mathrm{rect}\left(\frac{t}{T} - \frac{1}{2}\right) - \sqrt{2} \cdot \mathrm{rect}\left(\frac{t}{T} - \frac{3}{2}\right) \\
g(t) &= -\sqrt{2} \cdot \mathrm{rect}\left(\frac{t}{T} - \frac{1}{2}\right) + \sqrt{7} \cdot \mathrm{rect}\left(\frac{t}{T} - \frac{3}{2}\right)
\end{aligned}
$$

(f) Determine $z(t)$ and $z_T(t)$ for $i(t) = B(t) \cdot cos(\omega_x t)$ and $q(t) = \hat{i}(t)$

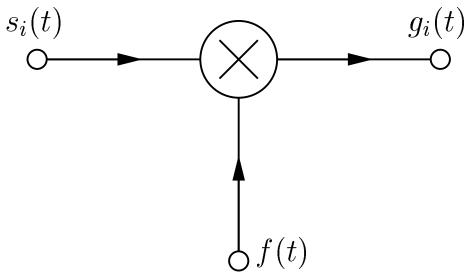
Exercise 18:

Examine the following systems on linearity and time-invariance:

(a) $g_i(t) = k + s_i(t)$

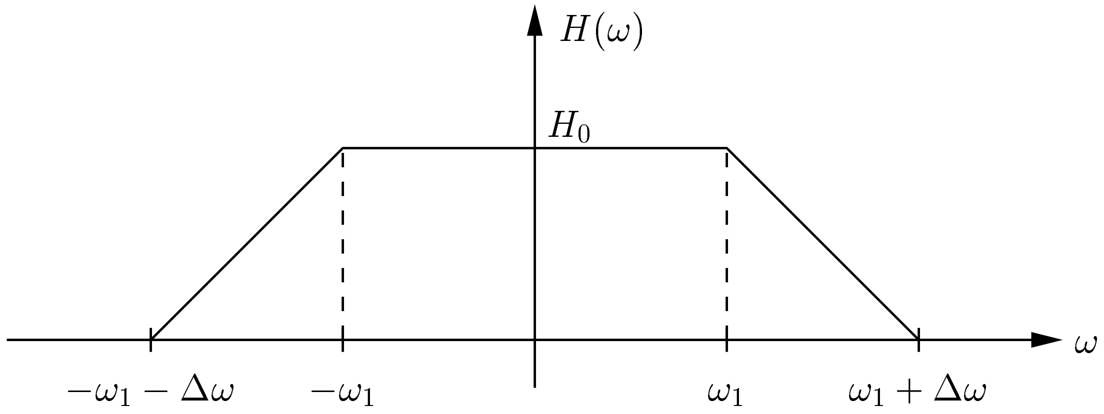


(b) $g_i(t) = f(t) \cdot s_i(t)$



Exercise 19:

Determine the impulse response of a linear, time-invariant transmission system with the sketched transfer function



Hint:
Represent the sketched transfer function as a convolution of two rect-functions and determine the impulse response by means of the standard table of Fourier transforms.

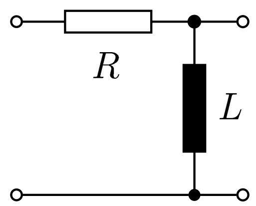
Exercise 20:

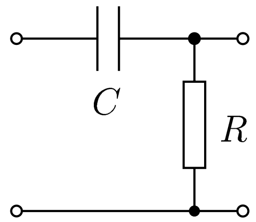
(a) Determine $h(t)$ for the RC-Lowpass. Note that this low-pass responds to

$$s(t) \;=\; \varepsilon(t) \quad \text{with} \quad u_c(t) = g(t) = \left(1 - e^{-\frac{t}{RC}}\right) \forall t \geq 0$$

(b) D̄escribe graphically the response of RC-Lowpass to rect $\left(\frac{t}{T} - 0,5\right)$
$with \tau = RC$ for $\tau \in \{0.2,\ 1.0,\ 5.0\} \cdot T$
$Note$:$\mathrm{e}^{-1} = 0.63$; $e^{-5} = 0.007$; $e^{-0.2} = 0.82$

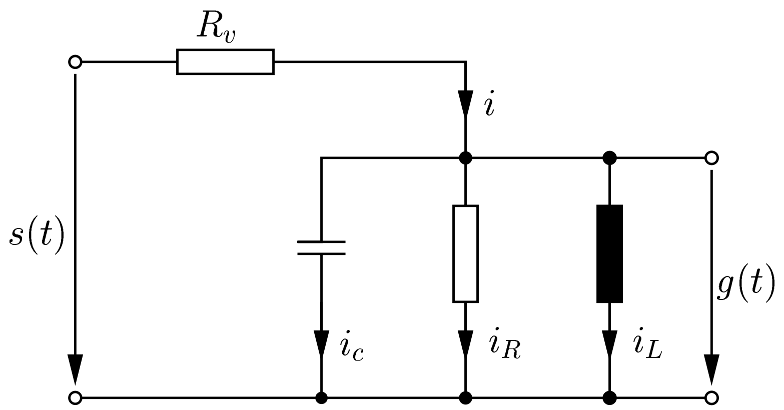
(c) Do the same for the RC-Highpass. Note: $u_R(t) + u_c(t) = s(t)$
(d) Determine $h(t)$ by means of Laplace-Transform of $s(t) = u_R(t) + u_c(t)$
(e) Determine $H(\omega)$ by means of network analysis and using impedances $Z_L = j\omega L$, $Z_c = \frac{1}{j\omega C}$
(f) Determine $H(\omega)$ for the following two networks (still using network analysis)





Exercise 21:

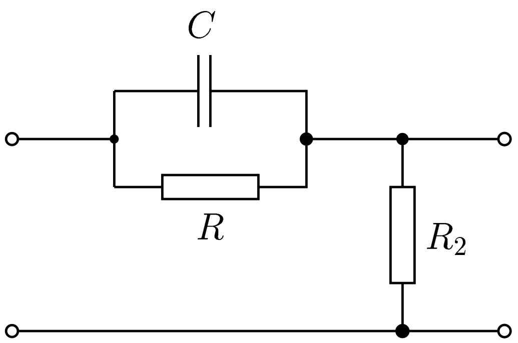
Given is the following schematic:



(a) Determine the differential equation for this system
(b) Determine the transfer function $H(\omega)$ and the system function $H_L(p)$

Exercise 22:

The following schematic is given:

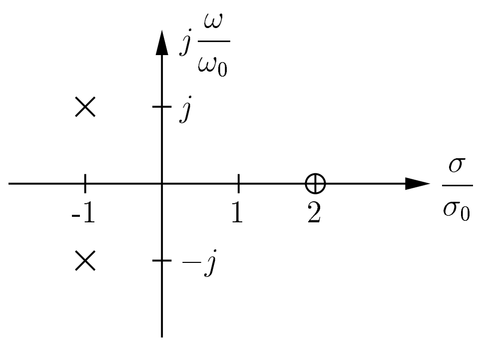


(a) Determine the transfer function $H(\omega)$
(b) Determine $|H(\omega)|$ for $\omega = 0, 1, \infty$ and $R = R_2$
(c) Determine for $\omega = 0, 1, \infty$ the damping measure $a_{dB}(\omega)$
(d) Determine for $\omega = 0, 1, \infty$ the damping phase $b(\omega)$
(e) Determine for $\omega = 0, 1, \infty$ the phase delay $\tau_{ph}\left(\frac{1}{RC}\right)$
(f) Determine for $\omega = 0, 1, \infty$ the group delay $\tau_{gr}(\omega)$

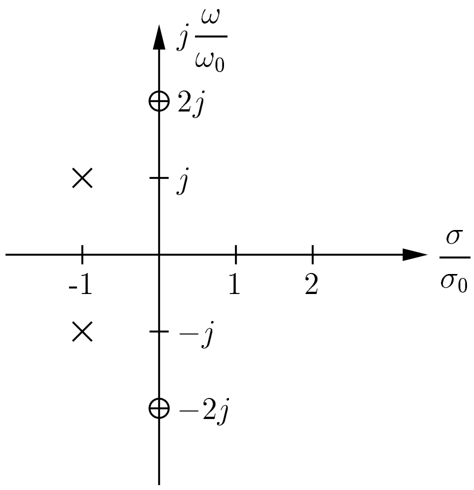
Exercise 23:

(a) Sketch $|H_L(j\omega)|$ for the following systems (S1) - (S5)
(b) Determine $H_L(p)$, apart from constant K
(c) Check stability

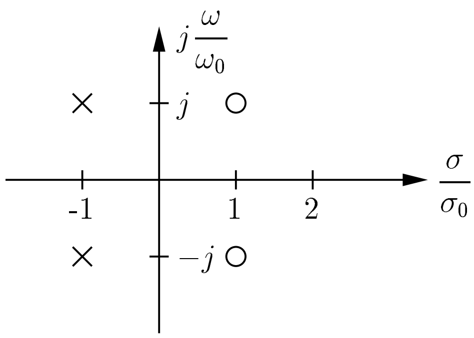
(S1)



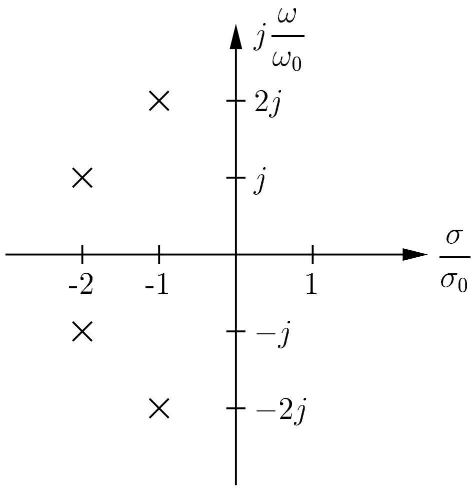
(S2)


$j\frac{\omega}{\omega_0}$
$2j$
j
$\frac{\sigma}{\sigma_0}$
-1
1
2
$-j$
$-2j$

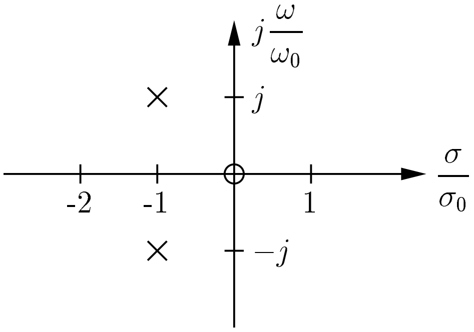
(S3)


$j\frac{\omega}{\omega_0}$
j
$\frac{\sigma}{\sigma_0}$
-1
1
2
$-j$

(S4)

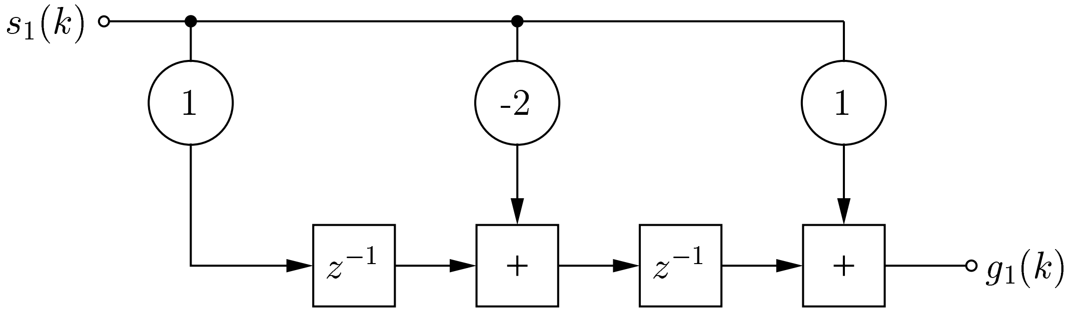

$j\frac{\omega}{\omega_0}$
$2j$
j
$\frac{\sigma}{\sigma_0}$
-2
-1
1
$-j$
$-2j$

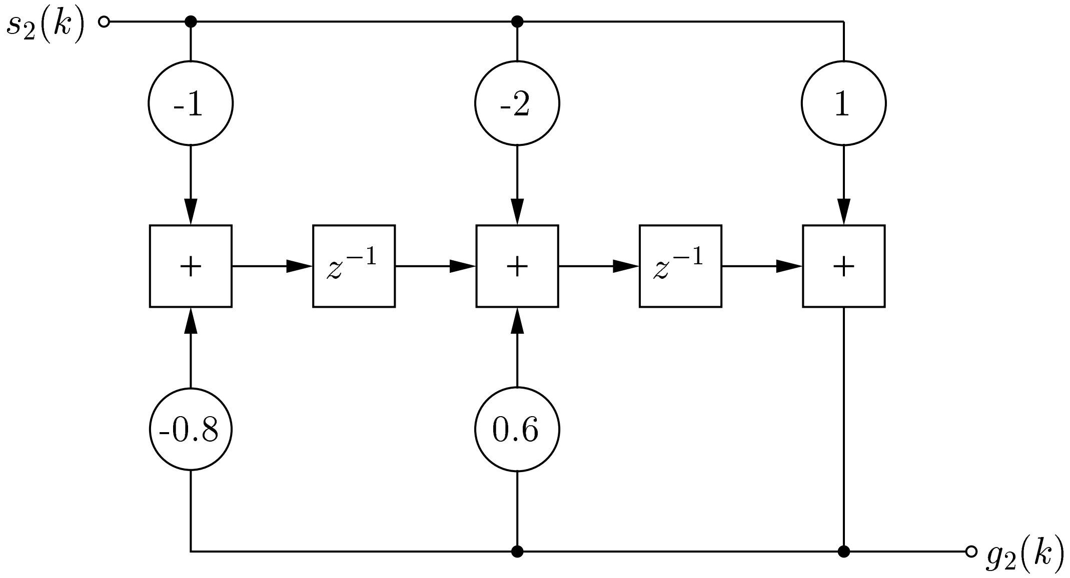
(S5)



Exercise 24:
Prove that conjugated poles/zeros are essential for ensuring real coefficients in $H_L(p)$

Exercise 25:
Given are the following systems:





(a) Determine the difference equation and the transfer function of both systems.
(b) Calculate the first 5 values of the impulse response for both systems.

(c) Determine the resulting transfer function for
   (c1) serial connection,
   (c2) parallel connection,
of both systems.

Exercise 26:

A discrete system computes the arithmetic mean value of the last two samples of the input signal.

(a) Write down the difference equation of the system and determine the transfer function $H_z(z)$ and the impulse response $h(k)$.
(b) Determine the amplitude response $|H(e^{j\omega T})|$ and sketch the function using all characteristic values.
(c) Carry out the same considerations as above for the general mean value filter with the following difference equation

$$g(k) = \frac{1}{N}\sum_{i=0}^{N-1} s(k-i) \quad \text{for} \quad N = 4$$

Exercise 27:

Given is a discrete system with the following difference equation:

$$g(k) = s(k-1) - \frac{1}{2} g(k-1)$$

Determine the impulse response $h(k)$

(a) by inverse z transform of the transfer function $H_z(z)$
(b) by inserting the unit pulse $\delta(k)$ in the difference equation
(c) Draw the signal flow diagram
(d) Determine the amplitude and phase frequency response $H_a(\omega)$ and $H_a'(\Omega)$ for $T = 1\mu s$

Exercise 28:

Given is a difference equation of a discrete system:

$$\begin{aligned} g(k) = \; & s(k) + 2.8 \cdot s(k-1) + 3.92 \cdot s(k-2) + 2.8 \cdot s(k-3) + s(k-4) + \\ & +3 \cdot g(k-1) - 3.62 \cdot g(k-2) + 2.062 \cdot g(k-3) - 0.4745 \cdot g(k-4) \end{aligned}$$

Determine the four canonical filter structures and their respective form of the transfer function.

Exercise 29:

The following difference equation is given:

$$5 \cdot s(k-1) + 2 \cdot s(k-2) \;=\; 2 \cdot g(k) + 16 \cdot g(k-1) + 50 \cdot g(k-2)$$

(a) Determine the transfer function $H_z(z)$ of the system
(b) Determine the poles and the zeros of the system
(c) Draw the corresponding signal flow diagram

Exercise 30:

Given are the following systems:

$$\text{I)} \quad H_{z1}(z) = \frac{z^2 + \frac{1}{9}}{z^2 - \frac{1}{4}} \qquad\qquad \text{II)} \quad H_{z2}(z) = \frac{z^2 - 4}{z^2 - 9}$$

(a) Which systems are stable ?
(b) Which are of minimum phase ?

Exercise 31:

Separate the following transfer functions into minimum phase systems and all-pass systems

$$H_{z1}(z) \;=\; \frac{(1 - 0.5 \cdot z) \cdot (1 - 0.25 \cdot z)}{(z - \frac{1}{3})(z - \frac{1}{5})}$$

$$H_{z2}(z) \;=\; \frac{(2 \cdot z + 1) \cdot (z + 2)}{(4 \cdot z + 1) \cdot (z + 4)}$$