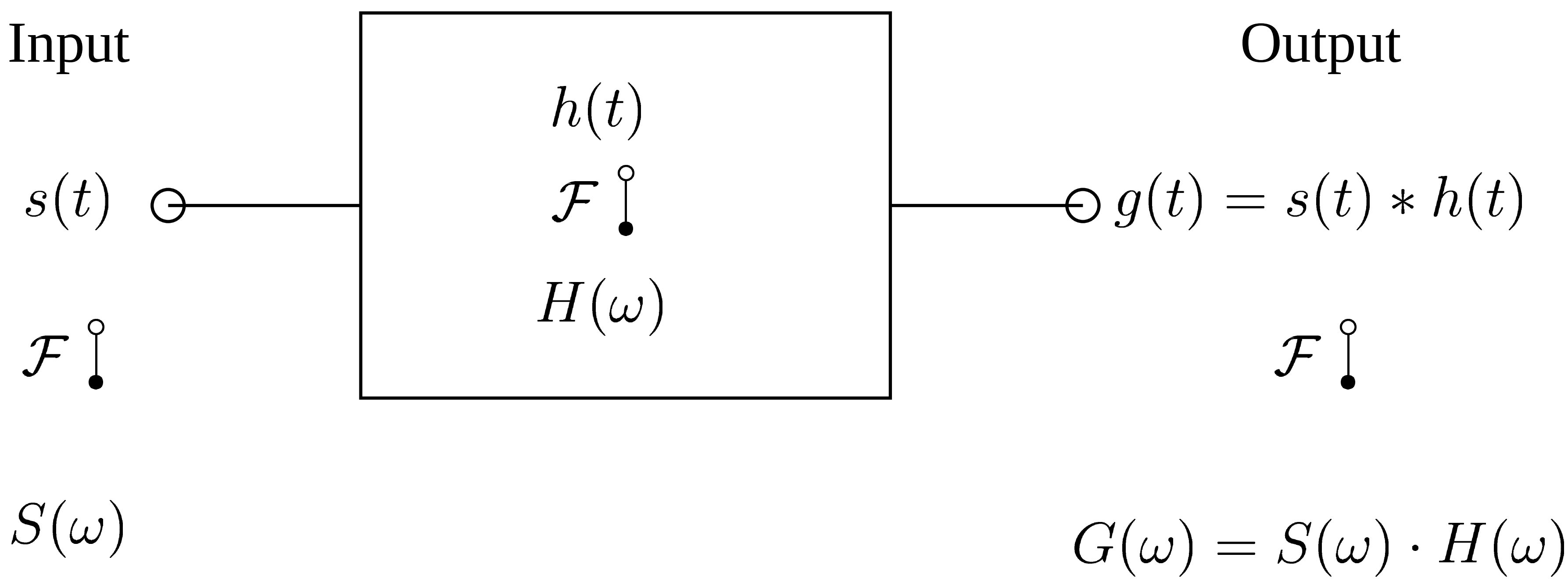
Seminar topic 1 “Linear time invariant systems”

Author: Rainer Siebel, Fachgebiet Nachrichtentechnik

Gerhard-Mercator-Universität Duisburg, Germany

1. “Black Box” consideration for LTI-systems.
2. Pulse response $h(t)$ and transfer function $H(\omega)$.
3. LTI-systems with sinusoidal input signals.
4. NTI-systems with sinusoidal input signals.
5. Idealizing simplifications and practical considerations.
6. Summary.

LTI-System



The pulse response $h(t)$ and its Fourier-transform $H(\omega)$ determine completely the transfer behaviour of LTI-systems.

- provided, the system is in its zero-state!

The pulse response $h(t)$ is the reaction of the LTI-system in case of an ideal Dirac-pulse $\delta(t)$ as input signal.

- It is impossible to realize a Dirac-pulse. Hence, the Dirac pulse and its corresponding constant spectrum are idealizations!
- For the determination of the pulse response in practice a pulse $\delta_a(t)$ with finite duration Δt and finite height A is sufficient, if the pulse duration is much shorter than the reciprocal $\frac{1}{f_c}$ of the upper cut-off frequency of the system under test.
- Apart from a vertical scale factor the corresponding approximated pulse response $h_a(t)$ is $\approx$ proportional to the theoretical pulse response, i.e. $h(t) \approx a \cdot h_a(t)$.
- The scale factor a depends on the shape of the approximated Dirac-pulse $\delta_a(t)$.

The transfer function $H(\omega)$ describes the transfer behaviour of an LTI-system in the frequency domain.

- i.e., the continuous or discrete spectrum $S(\omega)$ of the input signal is weighted according to $G(\omega) = S(\omega) \cdot H(\omega)$ to form the output spectrum $G(\omega)$.
- The transfer function apparently describes typical transfer characteristics like low-pass-, high-pass-, band-pass-, all-pass- or band-stop characteristics.
- Causality is the minimum requirement for a realizable system, i.e. $h(t) \equiv 0 \quad \forall t < 0$.
  From the causality requirement follows that real- and imaginary part of $H(\omega)$ are related to each other by the Hilbert transform!
  Moreover for *minimum phase systems*, the magnitude characteristic $|H(\omega)|$ determines uniquely the phase characteristic $\varphi(\omega)$.

For a sinusoidal input signal

$$s(t) = A \cdot \sin(\omega_0 t + \varphi_0) \overset{\mathcal{F}}{\circ\!\!-\!\!\bullet} A \cdot j\pi \left[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)\right] \cdot e^{j\omega \frac{\varphi_0}{\omega_0}}$$

LTI-systems respond with a sinusoidal output signal:

$$g(t) = A \cdot |H(\omega_0)| \cdot \sin(\omega_0 t + \varphi_0 + \varphi(\omega_0))$$

$$\mathcal{F} \updownarrow \qquad\qquad \mathcal{F} \updownarrow$$

$$G(\omega) = A \cdot |H(\omega_0)| \cdot j\pi \left[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)\right] \cdot e^{j\omega \frac{\varphi_0 + \varphi(\omega_0)}{\omega_0}}$$

- i.e. the magnitude of the output signal is proportional to the magnitude of the input signal with proportionality factor $|H(\omega_0)|$
- as compared to the input signal the output signal is delayed by the so called **phase delay time** $t_{\text{ph}} = -\dfrac{\varphi(\omega_0)}{\omega_0}$.
- We can use this feature for the determination of the transfer function in laboratory experiments!

Nonlinear time invariant systems respond to sinusoidal input signals with *non linear distortions*,

- i.e. the output signal is still periodic but no more sinusoidal
- and the output spectrum shows additional spectral components as compared to the two spectral lines of the input spectrum! (harmonic spectral components, distortion factor).

Using the Fourier series expansion for periodic signals:

$$g_{\text{NTI}}(t) = \sum_{k=0}^{\infty} d_k \cdot \cos(k \cdot \omega_0 t + \varphi_k) \qquad \text{we find:}$$

$$G_{\text{NTI}}(\omega) = \pi \cdot \sum_{k=0}^{\infty} d_k \cdot [\delta(\omega + k \cdot \omega_0) + \delta(\omega - k \cdot \omega_0)] \cdot e^{j\omega \frac{\varphi_k}{k \cdot \omega_0}}$$

- i.e., harmonic spectral components at whole-numbered multiple frequencies of ω_0!

Dealing with LTI-systems in the lectures and exercises we often ignore that the considered systems are not realizable,
(ideal low-pass-, high-pass, band-pass systems $\Rightarrow$ non-causal, etc.)

with the advantage, that the corresponding calculations are comparatively simple but nevertheless give us a deep insight to the transfer behaviour of compound systems!

The electronic circuit realization of such systems usually requires only

- a sufficiently *constant magnitude characteristic* in the transfer frequency range
- *and* in the same frequency range a sufficiently linear phase characteristic,

because in most cases we only need *linear systems without distortion* for band limited signals.

The LTI-systems theory is still

- the most important theory for modern communication engineering
- and the base for all extended considerations like for example the “statistical signal theory”, “signal detection methods” and “signal parameter estimation” etc.!
- The transfer behaviour for time continuous input signals can be determined with simple calculations
- moreover, LTI-systems theory is a good basis to later understand the theory of linear time discrete systems (e.g. digital filters).

Some of the most important features and conclusions in the context of LTI-systems have been discussed.